1. Let \( n \geq 2 \) and \( x_1, x_2, \ldots, x_k \) be points in \( \mathbb{R}^n \). Show that \( \mathbb{R}^n \setminus \{x_1, x_2, \ldots, x_n\} \) is path connected.

2. Show that \( \mathbb{R}^2 \setminus \mathbb{Q}^2 \) is path connected.

3. (a) Is a product of path-connected spaces necessarily path connected?
   (b) If \( A \subset X \) and \( A \) is path connected, is \( \overline{A} \) necessarily path connected?
   (c) If \( f : X \to Y \) is continuous and onto, and if \( X \) is path connected, does it necessarily follow that \( Y \) is path connected?
   (d) If \( \{A_\alpha\} \) is a collection of path connected spaces such that \( \cap A_\alpha \neq \emptyset \), is \( \cup A_\alpha \) necessarily path connected?

4. Show that if \( U \) is an open connected subspace of \( \mathbb{R}^n \), then \( U \) is path connected.

5. What are the connected components and the path components of \( \mathbb{R}^N \) in the product topology?

6. Show that a finite union of compact subspaces of a topological space \( X \) is compact.

7. Let \( A \) and \( B \) be compact subspaces of \( X \times Y \), and let \( N \) be an open set in \( X \times Y \) containing \( A \times B \). If \( A \) and \( B \) are compact, then there exist open sets \( U \) and \( V \) in \( X \), respectively \( Y \), such that

\[
A \times B \subset U \times V \subset N.
\]

8. Show that every compact subspace of a metric space is bounded and closed. Is the converse true?