1. Show that Example 1 from §2.1 describes indeed a closed set.

2. (a) Give an example of a continuous function $f : X \rightarrow Y$ for which there is an open set $U \subset X$ such that $f(U)$ is not open.
    (b) Give an example of $f : X \rightarrow Y$ such that there is a closed set $A \subset X$ for which $f(A)$ is not closed.
    (c) Give an example of a continuous map $f : X \rightarrow Y$ that is not a homeomorphism with the property that for every open set $U \subset X$, $f(U)$ is open and for every closed set $A \subset X$, $f(A)$ is closed.

3. Give an example of a bijective continuous function $f : X \rightarrow Y$ that is not a homeomorphism.

4. Prove Proposition 2.2.2.

5. Let $X$ be a topological space. The suspension $\Sigma X$ is defined as the quotient space $X \times [-1, 1]/\sim$ with the following equivalence relation:
   - For $\lambda \neq +1, -1$, $(x, \lambda) \sim (y, \mu)$ if and only if $x = y$ and $\lambda = \mu$.
   - $(x, 1) \sim (y, 1)$ for all $x, y \in X$.
   - $(x, -1) \sim (y, -1)$ for all $x, y \in X$.
   
   For the topological spaces $X = [0, 1]$ and $Y = S^1$ describe $\Sigma X$ and $\Sigma Y$. Prove that the suspension of a Hausdorff space is Hausdorff.

6. Given the topological spaces $X$ and $Y$ the join $X \ast Y$ is defined as the quotient space $X \times [-1, 1] \times Y/\sim$ with the equivalence relation given by
   - for $\lambda \neq +1, -1$, $(x, \lambda, y) \sim (x', \lambda', y')$ if and only if $x = x'$, $\lambda = \lambda'$, $y = y'$,
   - $(x, -1, y) \sim (x, -1, y')$ for all $x \in X, y, y' \in Y$,
   - $(x, 1, y) \sim (x', 1, y)$ for all $x, x' \in X, y \in Y$.
   
   Given $X = \{1, 2\}$, $Y = [0, 1]$ find $X \ast X$ and $X \ast Y$. Prove that the join of Hausdorff spaces is Hausdorff.