1. Define $D : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty)$ by
\[
D(x, y) = |x_1 - y_1| + |x_2 - y_2| + \cdots + |x_n - y_n|.
\]
Prove that $D$ is a metric and that it induces the standard topology on $\mathbb{R}^n$.

2. Show that $d : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow [0, \infty)$ defined by
\[
d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}
\]
is indeed a metric.

3. Consider the 2-dimensional torus $S^1 \times S^1$ as a subspace of $\mathbb{C}^2 = \mathbb{R}^4$.
Show that the subspace topology is the same as the quotient topology defined in Example 1 from §1.3.7.

4. Show that the sphere $S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$
is a 2-dimensional manifold. (Hint. Let $N = (0, 0, 1)$ and $S = (0, 0, -1)$ be the North and the South poles. Take the stereographic projections onto the equatorial plane $\pi_N : S^2 \setminus \{N\} \rightarrow \mathbb{R}^2$ and $\pi_S : S^2 \setminus \{S\} \rightarrow \mathbb{R}^2$, then use the maps $f_N = \pi_N^{-1} : \mathbb{R}^2 \rightarrow S^2$ and $f_S = \pi_S^{-1} : \mathbb{R}^2 \rightarrow S^2$.)

5. Show that, as a real 2-dimensional manifold, $\mathbb{C}P^1$ is homeomorphic to the sphere $S^2$. 