1. Axioms about Equality
   a. $\exists\, a, b \in \mathbb{R} \ni a \neq b$
   b. Reflexive: $(a \in \mathbb{R}) \Rightarrow (a = a)$
   c. Symmetric: $(a, b \in \mathbb{R}) \Rightarrow (a = b \Rightarrow b = a)$
   d. Transitive: $(a, b, c \in \mathbb{R}) \Rightarrow ((a = b$ AND $b = c) \Rightarrow (a = c))$

2. Field Axioms
   There exist two binary operations on $\mathbb{R}$, addition (+) and multiplication ($\cdot$) such that
   a. Commutative Law of Addition: $(a, b \in \mathbb{R}) \Rightarrow (a + b = b + a)$
   b. Associative Law of Addition: $(a, b, c \in \mathbb{R}) \Rightarrow (a + (b + c) = (a + b) + c)$
   c. Existence of Additive Identity: $(\exists\, 0 \in \mathbb{R} \ni \forall a \in \mathbb{R} \;\; a + 0 = a)$
   d. Existence of Additive Inverses: $(a \in \mathbb{R}) \Rightarrow (\exists\, -a \in \mathbb{R} \ni a + -a = 0)$
   e. Commutative Law of Multiplication: $a, b \in \mathbb{R} \Rightarrow a \cdot b = b \cdot a$
   f. Associative Law of Multiplication: $a, b, c \in \mathbb{R} \Rightarrow a \cdot (b \cdot c) = (a \cdot b) \cdot c$
   g. Existence of Multiplicative Identity: $(\exists\, 1 \in \mathbb{R} \ni \forall a \in \mathbb{R} \;\; a \cdot 1 = a)$
   h. Existence of Multiplicative Inverses: $(a \in \mathbb{R}$ AND $a \neq 0) \Rightarrow (\exists\, a^{-1} \in \mathbb{R} \ni a \cdot a^{-1} = 1)$
   i. Distributive Laws

3. Order Axioms
   a. There exist a set $\mathbb{R}^+ \subset \mathbb{R}$ such that
      a. Additive Closure: $(a,b \in \mathbb{R}^+) \Rightarrow (a+b \in \mathbb{R}^+)$
      b. Multiplicative Closure: $(a,b \in \mathbb{R}^+) \Rightarrow (a \cdot b \in \mathbb{R}^+)$
      c. Tricotomy: $(a \in \mathbb{R}) \Rightarrow (a \in \mathbb{R}^+$ OR $a = 0$ OR $-a \in \mathbb{R}^+)$!

4. Completeness Axiom (LUB Principle)
   a. $(S \subset \mathbb{R}$ and S bounded above)$\Rightarrow \exists\, x \in \mathbb{R} \ni x$ is a least upper bound for S

5. Consequences (Field Axioms)
   a. **Theorem**: $(a, b, c \in \mathbb{R}$ AND $a = b) \Rightarrow (a + c = b + c)$
   b. **Theorem**: $(a, b, c \in \mathbb{R}$ AND $a = b) \Rightarrow (ac = bc)$
   c. **Theorem**: $(a, b, c \in \mathbb{R}$ AND $a + c = b + c) \Rightarrow (a = b)$
   d. **Theorem**: $(a, b, c \in \mathbb{R}$ AND $ac = bc$ AND $c \neq 0) \Rightarrow (a = b)$
   e. **Theorem**: $(a, b \in \mathbb{R}$ AND $a + b = 0) \Rightarrow (a = -b$ AND $b = -a)$
   f. **Theorem**: $(a \in \mathbb{R}) \Rightarrow a \cdot 0 = 0$
   g. **Theorem**: $(a, b \in \mathbb{R}$ AND $a \cdot b = 0) \Rightarrow (a = 0$ OR $b = 0)$
   h. **Theorem**: $(a, b \in \mathbb{R}) \Rightarrow$
      a. $a = -(-a)$
      b. $-(a + b) = -a + -b$
      c. $(-a)b = -(ab)$
      d. $(-a)(-b) = ab$
   i. **Theorem**: $(a, b, c, d \in \mathbb{R}$ AND $b, d \neq 0) \Rightarrow (a/b = c/d \Leftrightarrow ad = bc)$
   j. **Theorem**: $(a, b, c \in \mathbb{R}$ AND $b, c \neq 0) \Rightarrow (ac / bc = a/b)$
   k. **Theorem**: $(a, b \in \mathbb{R}$ AND $a, b \neq 0$ AND $ab = 1) \Rightarrow (a = b^{-1}$ AND $b = a^{-1})$
   l. **Theorem**: $(a, b \in \mathbb{R}$ AND $a, b \neq 0) \Rightarrow$
      a. $a/a = 1$
      b. $a/1 = a$
      c. $1/a = a^{-1}$
      d. $1 / (1/a) = a$
      e. $(-a)/b = -\, a/b = a/(-b)$
      f. $(-a)/(-b) = a/b$
   m. **Theorem**: $(a, b, c, d \in \mathbb{R}$ AND $a, b, c, d \neq 0) \Rightarrow$
      a. $(1/a) \cdot (1/b) = 1/ (ab)$
      b. $(a/b) \cdot (c/d) = (ac) / (bd)$
      c. $a/c + b/c = (a+b) /c$
      d. $a/b + c/d = (ad + bc) / (bd)$
      e. $a/b - c/d = (ad-bc) / (bd)$
      f. $1 / (a/b) = b/a$
      g. $(a/b) / (c/d) = (a/b) \cdot (d/c)$

6. Consequences (Order Axioms)
   a. Order Properties
      a. $a \in \mathbb{R}^+ \Leftrightarrow -a \in \mathbb{R}_-$
      b. $a \in \mathbb{R}_- \Leftrightarrow -a \in \mathbb{R}^+$
   b. **Lemma**: $(a \in \mathbb{R}) \Rightarrow$
      a. $(a > 0) \Leftrightarrow (a \in \mathbb{R}^+)$
      b. $(a < 0) \Leftrightarrow (a \in \mathbb{R}_-)$
   c. **Lemma**: $(a, b \in \mathbb{R}) \Rightarrow$
      a. $a \leq a$
      b. $(a \leq b$ AND $b \leq a) \Leftrightarrow (a = b)$
   d. **Theorem**: $(a, b \in \mathbb{R}) \Rightarrow$
      a. $(a > 0$ AND $b < 0) \Rightarrow (ab < 0)$
      b. $(a < 0$ AND $b < 0) \Rightarrow (ab > 0)$
      c. $a^2 \geq 0$
   e. **Theorem**: $(a, b, c \in \mathbb{R}) \Rightarrow$
      a. $(a < b$ AND $b < c) \Rightarrow (a < c)$
      b. $(a < b) \Leftrightarrow (a + c < b + c)$
      c. $(a < b$ AND $c > 0) \Leftrightarrow (ac < bc$ AND $c > 0)$
      d. $(a < b$ AND $c < 0) \Leftrightarrow (ac > bc$ AND $c < 0)$