Notes: 17 Feb 2003

Sequences

A sequence is a (infinite) string of numbers in a specified order, e.g.,

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

The members of a sequences are referred to by their index in the sequence

In the above example, if we denote the sequence as $\{x_k\}$, then

$x_3 = 2$, which is the third number in sequence
$x_7 = 13$ which is the seventh number in sequence
etc.

Frequently sequences are frequently generated by formulas

Arithmetic sequences: each term is obtained from the preceding term by <u>adding</u> on a fixed amount

e.g. 3, 7, 11, 15, 19, 23, 27, 31, 35, . . .

Geometric sequences: each term is obtained from the preceding term by <u>mutliplying</u> by a fixed amount

e.g. 8, 4, 2, 1, 1/2, 1/4, 1/8, 1/16, . . .

Recursive sequences: each term is obtained from the previous terms by some specified formula involving the previous terms

e.g. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

here $x_1 = 1$
$x_2 = 1$
$x_k = x_{k-1} + x_{k-2}$, for $k > 2$

Note: Alternate notation for $x_k = x[k]$

Spreadsheets are particularly functional for generating sequences where new terms are obtained by using formulas on preceding terms because of the ease in copying such formulas from one cell to the next in either a column or row

Fractals

Geometric constructions which frequently exhibit self-similarity

Fern (in class)
Mandelbrot Set
Julia Sets
Sierpenksi Carpet

R^2 : Cartesian plane; every point P has a pair of coordinates (x,y)

Transformations (or Mappings or Functions) from R^2 to R^2 are made up of pairs of equations for assigning values (new ordered pairs) to each ordered pair in R^2

e.g.
$$u = 2x - 3y - 1$$
$$v = -x + y + 2$$

(1,2) (-8,3)
(0,1) (-4,3)
etc.

Linear Transformations are transformations for which the defining equations are linear. Linear transformations map straight lines to straight lines and straight line segments to straight line segments. Polygonal regions are mapped to polygonal regions because their bounding (straight) line segments are mapped to (straight) line segments.

In particular, the unit square (the square with vertices (0,0), (1,0), (1,1) and (0,1)) is mapped under a linear transformation to a quadralateral whose boundary is made up of the images of the four line segments which bound the unit square.

Contraction Mappings of the Unit Square are mappings which take every point in the unit square back into the unit square.

e.g.
$$u = 0.2x - 0.26y + 0.4$$
$$v = 0.23x + 0.22y + 0.045$$

Orbit of a point (x,y) is the set of all images of the point (x,y) under a sequence of transformations

(x,y) is mapped by first transformation to (x_1,y_1)
(x_1,y_1) is mapped by second transformation to (x_2,y_2)
(x_2,y_2) is mapped by third transformation to (x_3,y_3)
(x_3,y_3) is mapped by fourth transformation to (x_4,y_4)
etc.

Orbit = $\{ (x,y), (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), \ldots \}$

Fractals within the units square can be generated by

1. selecting a finite set of linear contractions of the unit square
2. selecting a starting point within the unit square, e.g. (½, ½)
3. computing the orbit of (x,y) where each succeeding point is generated by randomly selecting one of the linear contractions specified above (in step 1) and applying it to generate the next point in the sequence
4. plotting the generated points (in step 3)