# Homework Problems

**Conway, Functions of One Complex Variable**  
Spring 2014

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
<th>Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td>95</td>
<td>4-6, 8, 10-11</td>
</tr>
<tr>
<td>4.7</td>
<td>99</td>
<td>2-4, 6-7</td>
</tr>
<tr>
<td>4.8</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>P.1</td>
<td></td>
<td>Verify the parenthetical comment on page 98:</td>
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<tr>
<td></td>
<td></td>
<td>To show the second equality above takes a little effort, although for ( \gamma ) smooth it is easy. The details are left to the reader.</td>
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<tr>
<td>5.1</td>
<td>110</td>
<td>1a,b,c,e,h,j, 4, 6, 8, 10, 13-14, 16</td>
</tr>
<tr>
<td>5.2</td>
<td>121</td>
<td>1, a,c, 2,a,b,c,d, 3-4, 6</td>
</tr>
</tbody>
</table>
| P.2     |      | Show a) \[ \int_{0}^{\infty} \frac{x^{\alpha}}{1+2x+x^{2}} \, dx = \frac{\pi \alpha}{\sin \pi \alpha}, \quad -1 < \alpha < 1 \]  
|         |      | b) \[ \int_{0}^{\infty} \frac{dx}{x^{3}+1} = \frac{2\pi}{3\sqrt{3}} \] |
| 5.3     | 126  | 2, 6, 9-10 |
| P.3     |      | Show that the equation \( z = \exp(z) \) has an infinite number of solution, each of which lies in the region \( \text{Re } z \geq -1 \). |
| 6.1     | 129  | 1, 2, 6-7 |
| 6.2     | 132  | 1-3, 6-8 |
| P.4     |      | Show for \(|a| < 1, \, |b| < 1\) that  
|         |      | \[ \frac{|a|-|b|}{1-|a||b|} \leq \frac{|a+b|}{|1+ab|} \leq \frac{|a|+|b|}{1+|a||b|}. \]  
|         |      | When does equality occur? |
| 6.3     | None | |
| 6.4     | 140  | 3-4, 6 |
| 7.1     | 150  | 1-2, 4-5, 7-8 |
| 7.2     | 154  | 4, 6, 8, 10, 13 |
P. 5 Prove Alexander's Theorem: \( f \in K \) if and only if \( zf' \in S^* \)

P. 6 Prove: If \( f \in K \), \( f(z) = z + \sum_{n=0}^{\infty} a_n z^n \), then \( |a_n| \leq 1 \).

P. 7 Prove: If \( f \in K \), then \( f(D) \supset D_{1/2} \)

P. 8 Prove: If \( f \in S^* \), then there exists a probability measure \( \mu \in P(\partial D) \) such that

\[
f(z) = z \exp\left(-2\int_0^z \log(1 - xz) d\mu(x)\right)
\]

P. 9 Show that \( \Gamma \) has exactly one absolute minimum point on \( \{ x \mid x > 0 \} \) and that it lies in the interval \((1, 2)\).

P. 10 Let \( F(z) = \Gamma(z) - \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(z+n)} \), \( z \in \mathbb{C} \). Show that \( F \) is an entire function and that

\[
F(z) = \int_1^\infty t^{z-1}e^{-t} dt, \, \text{Re} z > 1
\]