MATH 5321 Exam I 3 March 2013

Make-up

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. X

2. X

3. (21 pts) Identify each of the finite isolated singularities for each of the following functions. Determine whether the singularities are removable singularities, poles or essential singularities. If the singularity is removable, determine what value the function should be assigned at the singularity to analytically extend the function at the singularity. If the singularity is a pole, determine the order of the pole.

a. $f(z) = \dfrac{e^{\pi z} + 1}{z^4 - 1}$ b. $f(z) = \dfrac{1}{\cos(2z) - 1} + \dfrac{1}{2z^2}$

c. $f(z) = (1 - z^4)\cos\dfrac{1}{z^2}$ d. $f(z) = \dfrac{z^3 - 2z^2 + z}{z(1-z)^4(2-z)^3}$

4. (21 pts) Each of the following functions has exactly one isolated singularity inside the disk $B(1;\frac{5}{4})$. Determine the reside of the function at that isolated singularity.

a. $f(z) = \dfrac{\sqrt{3z}}{e^{2iz} + e^{-2iz}}$ b. $f(z) = \dfrac{(1-z^2)^2 e^{-1/z}}{4z}$

c. $f(z) = \dfrac{\left(\log(z)\right)^2}{(z-1)^4}$

5. (20 pts) Prove the following theorem about the integration of rational functions:

Theorem 6. Let r be a rational function such that $r = p/q$ where p is a even polynomial with real coefficients of degree two, q is an even polynomial with real coefficients of degree four, and p and q are relatively prime. Suppose q has exactly one root in Q_1, say at $z_1 = a + bi$ where $a > 0$ and $b > 0$. Then,

i. The roots of q are $z_1,\ \overline{z_1},\ -z_1,\ -\overline{z_1}$

ii. $res(r;-\overline{z_1}) = -res(r,\overline{z_1})$

iii. $\int_0^\infty r(x)dx = \pi i\left[res(r;z_1)+res(r;-\overline{z_1})\right]$

$= \pi i\left[res(r;z_1)-res(r;-\overline{z})\right]$

Note: The first equality in iii. is a consequence of Theorem 1 of the theorems on applications of the residue theorem to the integration of rational functions.

6. (12 pts) Classify each of the following regions at to whether they are convex or not convex, starlike (with respect to some point $a \in G$) or not starlike (with respect to any point $a \in G$) , simply connected or not simply connected.

N.B. Mark each cell in the table as Y or N, i.e., do not leave any cell in the table blank.

Notation:

- (a,b) denotes the (open) straight line segment between a and b
- [a,b] denotes the (closed) straight line segment between a and b.
- $((a_1,\ a_2,\ \cdots,\ a_n))$ denotes the (open) interior of the polygon with vertices $a_1,\ a_2,\ \cdots, a_n$.

Use the table on the next page to report your answers

a. $G_a = B(0,1)\setminus\{(-1,-\tfrac{1}{2}]\cup[\tfrac{1}{2}i,i)\}$

b. $G_b = B(0,1)\setminus\{(-1,-\tfrac{1}{2}]\cup[-\tfrac{1}{2}i,i)\}$

c. $G_c = B(-2,3)\cap B(9,10)\cap B(6i,7)\cap B(-11i,12)$

d. $G_d = B(0,4)\setminus\overline{B(7,10)}$

e. $G_e = B(0,1)\cup B(\tfrac{1}{2},1)\cup B(1,1)$

f. $G_f = ((-2+i,1+i,1))\cup((-1,1+i,1))$

g. $G_g = ((-2+i,2+i,2-i,-2-i))\cup B(i,2)\cup B(2,1)$

h. $G_h = B(0,10)\setminus\{[-\tfrac{1}{2},\tfrac{1}{2}]\cup[-\tfrac{1}{2}i,\tfrac{1}{2}i]\}$

Name ______________________________ (Return this sheet with your other problems)

	Convex	Starlike	Simply Connected		Convex	Starlike	Simply Connected
G_a				G_e			
G_b				G_f			
G_c				G_g			
G_d				G_h			