MATH 5321 Exam I 26 February 2014

Take Home

Due: 3 March 2014

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (12 pts) Find the Laurent series expansion for $f(z)=\dfrac{z^2-z+7}{(z-2)^2(z+1)}$ on:

(a) $ann(0;1,2)$; (b) $ann(-1;0,3)$; (c) $ann(-1;3,\infty)$; (d) $ann(i;\sqrt{2},\sqrt{5})$

2. (15 pts) Identify each of the finite isolated singularities for each of the following functions. Determine whether the singularities are removable singularities, poles or essential singularities. If the singularity is removable, determine what value the function should be assigned at the singularity to analytically extend the function at the singularity. If the singularity is a pole, determine the order of the pole and, if the pole occurs at z = 0, then determine the singular part of the function at 0.

(a) $f(z)=\dfrac{z^2+1}{z^2(z^2-1)}$ (b) $f(z)=\dfrac{\sin(z^2)}{z(\exp(iz)-1)}$

(c) $f(z)=\dfrac{\sin(2\pi z)}{z^4(1+z^2)}$ (d) $f(z)=\dfrac{\sin(2\pi z)}{z(1+z^3)}$

(e) $f(z)=\dfrac{z}{\exp(1/z)-1}$

3. (8 pts) Each of the following functions has a pole at z = 0. Determine the order of the pole at z = 0 and the determine the residue of the function at z = 0.

(a) $f(z)=\dfrac{z^3\sin(2z^4)}{\cos(z^4)-1}$ (b) $f(z)=\dfrac{(z\sin(4z))^2}{(1-\cos(2z^2))^2}$

4. (16 pts) Compute the value of each of the following integrals using the Residue Theorem:

a. $\displaystyle\int_{-\infty}^{\infty}\frac{x+1}{x^4+x^2+1}\,dx$ b. $\displaystyle\int_{0}^{\infty}\frac{\sin x}{x^5+2x^3+x}\,dx$

For each integral justify that your computation is a valid application of an appropriate theorem.