Homework Problems
Conway, *Functions of One Complex Variable*
Spring 2009

4.6 Page 95 4-6, 8, 10-11

4.7 Page 99 2-4, 6-7

4.8 None

P. 1. Verify the parenthetical comment on page 98:

> To show the second equality above takes a little effort, although for γ smooth it is easy. The details are left to the reader.

5.1 Page 110 1a,b,c,e,h,j, 4, 6, 8, 10, 13-14, 16

5.2 Page 121 1,a,c, 2,a,b,c,d, 3-4, 6

P.2 Show a) $\int_0^\infty \frac{x^\alpha}{1+2x+x^2}\,dx = \frac{\pi\alpha}{\sin \pi\alpha},\ -1<\alpha<1$

b) $\int_0^\infty \frac{dx}{x^3+1} = \frac{2\pi}{3\sqrt{3}}$

5.3 Page 126 2, 6, 9-10

P.3 Show that the equation $z=\exp(z)$ has an infinite number of solution, each of which lies in the region $\operatorname{Re} z \geq -1$.

6.1 Page 129 1, 2, 6-7

6.2 Page 132 1-3, 6-8

P.4 Show for $|a|<1$, $|b|<1$ that $\frac{|a|-|b|}{1-|a||b|} \leq \frac{|a\pm b|}{|1\pm \bar{a}b|} \leq \frac{|a|+|b|}{1+|a||b|}$.

When does equality occur?

6.3 None

6.4 Page 140 3-4, 6

7.1 Page 150 1-2, 4-5, 7-8

7.2 Page 154 4, 6, 8, 10, 13

7.4	Page 163	4-7

P. 5 Prove Alexander's Theorem: $f \in K$ if and only if $zf' \in S^*$

P. 6 Prove: If $f \in K$, $f(z) = z + \sum_{n=0}^{\infty} a_n z^n$, then $|a_n| \le 1$.

P. 7 Prove: If $f \in K$, then $f(D) \supset D_{1/2}$

P. 8 Prove: If $f \in S^*$, then there exists a probability measure $\mu \in \mathcal{P}(\partial D)$ such that

$$f(z) = z \exp\left(-2\int_0^z \log(1 - xz)\, d\mu(x) \right)$$

7.5	Page 173	4-7, 9
7.6	Page 176	1
7.7	Page 185	1-3, 7, 8

P. 9 Show that Γ has exactly one absolute minimum point on $\{x \mid x > 0\}$ and that it lies in the interval (1,2).

P. 10 Let $F(z) = \Gamma(z) - \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(z+n)}$, $z \in \mathbb{C}$ Show that F is an entire function and that $F(z) = \int_1^{\infty} t^{z-1} e^{-t}\, dt$, $\operatorname{Re} z > 1$

7.8	Page 194	1-2
8.1	Page 201	2
8.2	None	
8.3	None	
9.1	None	
10.1	Page 255	1-2, 4-7
10.2	None	
10.3	Page 268	1