MATH 5321 Exam II 30 March 2009

Take Home

Due: 6 April

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

1. How many roots does $e^z = (e+1)z^5$ inside $|z| = 1$?

2. Show that the equation $z + 3 + 2e^z = 0$ has exactly one solution in the left half-plane.

3. Show that four of the roots of $z^5 + 15z + 1 = 0$ belong to $\operatorname{ann}(0; 3/2, 2)$.

4. Find the number of roots in the first quadrant of $p(z) = z^4 + z^3 + 5z^2 + 2z + 4$.

5. Let D denote the open unit disk (centered at 0) and let RHP denote the right half-plane. Let $F = \{ f \in A(D) : f : D \to RHP,\ f(0) = 4 \}$. Find the value of $\alpha = \max_{f \in F} |f'(0)|$.

6. Suppose that u is harmonic on $\mathbb{C}$ and that for each $z \in \mathbb{C}$ that $|u(z)| > 1$. Prove that u is constant.

7. Let Ω and G be regions in $\mathbb{C}$. Let $f \in A(\Omega)$ such that $f(\Omega) \subset G$ and let u be harmonic on G. Show that $u \circ f$ is harmonic on Ω.

8. Let f be an entire function such that (i) $f(0) = 4+3i$ and (ii) $|f(z)| \le 5$ for $|z| \le 1$. Find $f'(0)$.

9. Consider the following two "theorems". One is true and one is false.

   A. Let G be a region in $\mathbb{C}$ and let F be a normal subset of $A(G)$. Let $F' = \{ f' : f \in F \}$. Then, F' is normal.

   B. Let G be a region in $\mathbb{C}$ and let F be a normal subset of $A(G)$. Let $\grave{F} = \{ f : f' \in F \}$. Then, $\grave{F}$ is normal.

   Determine which is true and prove it. Give a counter-example to show that the other is false.