Homework Problems

Conway, *Functions of One Complex Variable*

Spring 2005

4.4 None

4.5 Page 87 1, 3, 4, 6-9

4.6 Page 95 4-6, 8, 10-11

4.7 Page 99 2-4, 6-7

4.8 None

P. 1. Verify the parenthetical comment on page 98:

> To show the second equality above takes a little effort, although for g smooth it is easy. The details are left to the reader.

5.1 Page 110 1a,b,c,e,h,j, 4, 6, 8, 10, 13-14, 16

5.2 Page 121 1,a,c, 2,a,b,c,d, 3-4, 6

P.2 Show a) $$\int_0^\infty \frac{x^a}{1+2x+x^2}\,dx = \frac{pa}{\sin pa},\ -1<a<1$$

b) $$\int_0^\infty \frac{dx}{x^3+1} = \frac{2p}{3\sqrt{3}}$$

5.3 Page 126 2, 6, 9-10

P.3 Show that the equation $z = \exp(z)$ has an infinite number of solution, each of which lies in the region $\operatorname{Re} z \ge -1$.

6.1 Page 129 1, 2, 6-7

6.2 Page 132 1-3, 6-8

P.4 Show for $|a| < 1$, $|b| < 1$ that $\dfrac{|a|-|b|}{1-|a||b|} \le \dfrac{|a \pm b|}{|1 \pm \bar{a}b|} \le \dfrac{|a|+|b|}{1+|a||b|}$.

When does equality occur?

6.3 None

6.4 Page 140 3-4, 6

7.1 Page 150 1-2, 4-5, 7-8

7.2 Page 154 4, 6, 8, 10, 13

7.4 Page 163 4-7

P. 5 Prove Alexander's Theorem: $f \in K$ if and only if $zf' \in S^*$

P. 6 Prove: If $f \in K$, $f(z) = z + \sum_{n=0}^{\infty} a_n z^n$, then $|a_n| \leq 1$.

P. 7 Prove: If $f \in K$, then $f(D) \supset D_{1/2}$

P. 8 Prove: If $f \in S^*$, then there exists a probability measure $m \in P(\partial D)$ such that

$$f(z) = z \exp\left(-2 \int_0^z \log(1 - xz)\, dm(x) \right)$$

7.5 Page 173 4-7, 9

7.6 Page 176 1

7.7 Page 185 1-3, 7, 8

P. 9 Show that Γ has exactly one absolute minimum point on $\{ x \mid x > 0 \}$ and that it lies in the interval (1,2).

P. 10 Let $F(z) = \Gamma(z) - \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(z+n)}$, $z \in \mathbb{C}$ Show that F is an entire function and that

$$F(z) = \int_1^{\infty} t^{z-1} e^{-t}\, dt, \mathrm{Re}\, z > 1$$

7.8 Page 194 1-2

8.1 Page 201 2

8.2 None

8.3 None

9.1 None

10.1 Page 255 1-2, 4-7

10.2 None

10.3 Page 268 1