MATH 5321 Exam II April 13, 2005

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (25 pts) State and prove Schwarz's Lemma.

2. (25 pts) Give the definition for each of the following:

   a. Let f have an isolated singularity at $z = a$. Then the residue of f at $z = a$ is . . .

   b. Let G be a region and let $f : G \to \mathbb{R}$. Let $a \in \partial G$ or $a = \infty$. Then,

   $\limsup_{z \to a} f(z) = \ldots$

   c. The Poisson kernel $P_r(q) = \ldots$

   d. A set $\mathcal{F} \subset \mathcal{C}(G, \Omega)$ is normal . . .

   e. A set $\mathcal{F} \subset \mathcal{C}(G, \Omega)$ is equicontinuous on a set $E \subset G$ if . . .

3. (25 pts) Show that exactly four of the roots of $z^5 + 15z + 1 = 0$ lie in the annulus $\text{ann}(0; \frac{3}{2}, 2)$.

4. (25 pts) Let G be a bounded region in $\mathbb{C}$.

   a. Let $\{f_n\} \subset \mathcal{C}(\overline{G}, \mathbb{C}) \cap \mathcal{A}(G)$ and let $f \in \mathcal{C}(\overline{G}, \mathbb{C}) \cap \mathcal{A}(G)$. Suppose that $f_n \to f$ uniformly on ∂G. Show that $f_n \to f$ in $\mathcal{C}(G, \mathbb{C})$.

   b. Give an example of a sequence $\{g_n\} \subset \mathcal{C}(\overline{G}, \mathbb{C})$ and a function $g \in \mathcal{C}(\overline{G}, \mathbb{C})$ such that $g_n \to g$ uniformly on ∂G but g_n does not converge to g in $\mathcal{C}(G, \mathbb{C})$.