

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (6 pts) Give an example of a region  $G$  such that: (i)  $G$  is starlike with respect to 0 **and** (ii) there does not exist a point  $a \neq 0$  such that  $G$  is starlike with respect to  $a$
2. (15 pts) On page 88, it is shown that “ $\sim$ ” is an equivalence relation. Give examples of open sets  $G$  such that
  - (a) for  $G$  the number of equivalence classes under “ $\sim$ ” is exactly 1
  - (b) for  $G$  the number of equivalence classes under “ $\sim$ ” is exactly 2
  - (c) for  $G$  there are an infinite number of equivalence classes under “ $\sim$ ”
3. (30 pts) Find the Laurent series expansion for  $f(z) = \frac{8 + z^2}{z^3 - z^2 - 2z}$  on:
  - (a)  $ann(0; 1, 2)$ ; (b)  $ann(1; 0, 1)$ ; (c)  $ann(1; 1, 2)$ ;
  - (d)  $ann(i; \sqrt{2}, \sqrt{5})$ ; (e)  $ann(i; \sqrt{5}, \infty)$
4. (30 pts) Identify each of the isolated singularities for each of the following functions. Determine whether the singularities are removable singularities, poles or essential singularities. If the singularity is removable, determine what value the function should be assigned at the singularity to analytically extend the function at the singularity. If the singularity is a pole, determine the order of the pole and, if the pole occurs at  $z = 0$ , then determine the singular part of the function at 0. ~~If the singularity is an essential singularity, determine the residue of the function at the singularity.~~
  - (a)  $f(z) = \frac{z^2 + 1}{z^2(z^4 - 1)}$
  - (b)  $f(z) = \frac{\log(1 + z)}{z(\exp(iz) - 1)}$
  - (c)  $f(z) = \frac{\sin(\frac{2\pi}{z})}{(z^4 - 1)^2}$
5. (14 pts) Each of the following functions has a pole at  $z = 0$ . Determine the order of the pole at  $z = 0$  and the determine the residue of the function at  $z = 0$ .
  - (a)  $f(z) = \frac{z^3 \sin(2z^3)}{\cos(z^4) - 1}$
  - (b)  $f(z) = \frac{(z \sin(z))^3}{(1 - \cos(z^2))^2}$
6. (6 pts) Find all possible values of  $I(\mathbf{g}) = \int_{\mathbf{g}} \frac{dz}{4 - z^2}$  where  $\mathbf{g}$  is any simple (non-self intersecting) rectifiable curve in  $\mathbb{C} \setminus \{-2, 2\}$  connecting 0 to 1.