In-Class Examination — Math 5321 — March 30, 1998

100 Points

1. Let $p(z) = 3z^3 - 2z^2 + 3iz - 9$. Prove that all of the roots of p lie in the annulus ann(0,1,2).

2. State and prove Rouche's Theorem.

3. Let G be a region and let u be harmonic on G. Prove (without using the maximum theorem for harmonic functions): If there exists an $a \in G$ such that $u(a) \le u(z)$ for all $z \in G$, then u is constant on G.

4. Suppose $\{f_n\}$ is a sequence in $C(G,\mathbb{C})$ such that $f_n \to f$. Suppose $\{z_n\}$ is a sequence in G such that $z_n \to z$. Prove that $f_n(z_n) \to f(z)$.