

Due: February 16

100 Points

1. Compute the following integrals (justify your results):

a. $\int_0^{\infty} \frac{x}{x^4 + x^2 + 1} dx$ b. $\int_0^{\infty} \frac{\sin x}{x^5 + 2x^3 + x} dx$

c. $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ d. $\int_0^{\infty} \frac{\log x}{x^4 + 1} dx$

2. Find the Laurent series expansion for $f(z) = (1 + z^2)/(z^4 - 2z^2 + 4z)$ on:
(a) ann(0, $\sqrt{2}$, 2); (b) ann(0, 2, ∞); (c) ann(-2, 2, $\sqrt{10}$); (d) ann(2, $\sqrt{2}$, 2).

3. Identify each of the isolated singularities for each of the following functions. Determine whether the singularities are removable singularities, poles or essential singularities. If the singularity is removable, determine what value the function should be assigned at the singularity to analytically extend the function at the singularity. If the singularity is a pole, determine the order of the pole and the singular part of the function at the singularity. If the singularity is an essential singularity, determine the residue of the function at the singularity.

(a) $f(z) = \frac{z^2 + 2z}{(z + 1)^2(z^2 - 4)}$ (b) $g(z) = \frac{e^{1/z}}{1 + z}$

4. Find all possible values of $I(\gamma) = \int_{\gamma} \frac{dz}{1 + z^2}$ where γ is any simple (non-self intersecting) rectifiable curve in $\mathbb{C} \setminus \{-i, i\}$ connecting 0 to 1.