Review Exam I

Complex Analysis

Underlined Propositions or Theorems: Proofs May Be Asked for on Exam

Chapter 1.

Real Numbers $(\mathbb{R},+,\bullet)$

Complete Ordered Field

Complex Numbers $(\mathbb{C},+,\bullet)$

Complete Field

Rectangular Form $z = (a,b) = a + bi$

Geometric Interpretation of Addition as Vector Addition

Polar Form $z = (r,q\ ) = r \text{ cis } q$

Geometric Interpretation of Multiplication via Polar Form

Operations on Complex Numbers and Interrelationships between Operators

$\operatorname{Re} z$ $\quad \operatorname{Im} z$ $\quad |z|$ $\quad \bar{z}$ $\quad z\text{-}w$ $\quad \dfrac{z}{w}$ $\quad \operatorname{Re} \bar{z}$ $\quad \operatorname{Im} \bar{z}$

$\operatorname{Re}(z+w)$ $\quad \operatorname{Im}(z+w)$ $\quad |z+w|$ $\quad \overline{z+w}$

$\operatorname{Re}(z\text{-}w)$ $\quad \operatorname{Im}(z\text{-}w)$ $\quad |z\text{-}w|$ $\quad \overline{z-w}$

$\operatorname{Re}(zw)$ $\quad \operatorname{Im}(zw)$ $\quad |zw|$ $\quad \overline{z-w}$

$\operatorname{Re}(\dfrac{z}{w})$ $\quad \operatorname{Im}(\dfrac{z}{w})$ $\quad |\dfrac{z}{w}|$ $\quad \overline{\dfrac{z}{w}}$

<u>Triangle Inequality, Reverse Triangle Inequality</u>

DeMoivre’s Formula, Integral Roots of a Complex Number, Roots of Unity

Analytical Geometry in Complex Plane

Forms of the equation of a line / half-plane

Forms of the equation of a circle / disk

$\mathbb{C}_\infty$ and Stereographic Projection on S

Projection and Reverse Projection Formulas for $z \in \mathbb{C}$ and $Z \in S$

Distance Formula for Points in $\mathbb{C}_\infty$

Chapter 2.

Definition of Metric Space

    <u>Examples</u> of Metric Spaces

    <u>Problems</u> about Determining Whether a Pair (X,d) is a Metric Space

Definition of Open/Closed Sets in Metric Space

    <u>Examples</u> of Open/Closed Sets

    <u>Problems</u> about Determining Whether a Set is Open/Closed

<u>Propositions 1.9 & 1.11</u> Construction of New Open/Closed Sets from Existing Open/Closed Sets

Definitions of Auxiliary Terms

    int A, $\overline{A}$, ∂A, dense

<u>Proposition 1.13</u>

Definition of Connected Metric Space

    <u>Examples</u> of Connected Metric Spaces

    <u>Problems</u> about Determining Whether a Pair (X,d) is a Connected Metric Space

Proposition 2.2 The only connected subsets of $\mathbb{R}$ are intervals

Theorem 2.3 Let G be an open subset of $\mathbb{C}$. Then, G is connected if and only if G is polygonally path connected.

Definitions of Component, Region

Propositions about Decomposition of Metric Spaces into Components

<u>Theorem 2.9</u>

Definition of Convergence of a Sequence in a Metric Space

    <u>Examples</u> of Convergent Sequences

    <u>Problems</u> about Determining Whether a Sequence is Convergent

Definition of Limit Point

<u>Propositions 3.2 & 3.4</u> (Extensions of Alternate Criteria in Proposition 1.13)

Definition of Cauchy Sequence and Proposition: $\mathbb{C}$ is complete

Cantor's Theorem

    Standard Examples Showing the Requirement of All Three Cantor Conditions for Validity of the Conclusion of Cantor's Theorem

Definition of Compact Metric Space

    <u>Examples</u> of Compact Metric Spaces

    <u>Problems</u> about Determining Whether a Metric Space is Compact

<u>Proposition 4.3</u>

Proposition 4.4

Proposition: Every compact metric space is complete

Proposition: If X is compact, then every infinite subset of X has a limit point in X

Definition of Sequential Compactness and Lebegue's Covering Lemma

Theorem 4.9; Heine-Borel Theorem

Definition for Mapping $f : X \to \Omega$ of $\lim_{x \to a} f(x) = a$

- Examples of Limits
- Problems about Determining Whether a Limit Exists

Definition for Mapping $f : X \to \Omega$ of Continuity of f at $x=a$ and of Continuity of f on X

- Examples of Continuous Functions
- Problems about Determining Whether a Function is Continuous

Propositions 5.2 & 5.3 Equivalent ways to characterize continuity at a point and on a space

Proposition 5.4 & 5.5 Algebra of Continuous Functions on a Space X and Continuity of Composition of Continuous Functions

Corollary Every Polynomial Function is Continuous on $\mathbb{C}$. Every Rational Function is Continuous on $\mathbb{C}$, where defined.

Definition of Uniformly Continuous Functions

- Examples of Uniformly Continuous Functions
- Problems about Determining Whether a Function is Uniformly Continuous

Definition of distance from a point to a set $d(x,A)$ and distance from a set to a set $d(A,B)$

Proposition 5.7

Theorem 5.8 Consequences of Continuity

Propositions 5.9, 5.10, 5.11, 5.13, 5.14 Consequences of Continuity

Theorem 5.15 $f : X \to \Omega$, f continuous on X, X compact implies f is uniformly continuous on X.

- Examples which show that if f is not continuous on X or if X is not compact, then f need not be uniformly continuous on X

Theorem 5.17

Definition of Sequence of Uniformly Converging Functions on a Set X

- Examples of Sequences of Uniformly Converging Functions
- Problems about Determining Whether a Sequence of Function is Converges Uniformly

Theorem 6.1

Weierstrass M-Test