1. (15) Discuss the convergence of three of the following series, i.e., determine where the series converge absolutely and where the series converge uniformly.

   a. \( \sum_{n=1}^{\infty} n^2 z^n \)  
b. \( \sum_{n=1}^{\infty} \frac{1 + \cos \frac{\pi}{n}}{n} z^n \)  
c. \( \sum_{n=1}^{\infty} \frac{1}{z^n + n^2} \)  
d. \( \sum_{n=1}^{\infty} n^{-z} \).

2. (9) Suppose that the radius of convergence of \( \sum_{n=0}^{\infty} a_n z^n \) is \( r \), where \( 0 < r < \infty \). Discuss the radius of convergence of each of the following series:

   a. \( \sum_{n=0}^{\infty} a_n z^{2n} \)  
b. \( \sum_{n=0}^{\infty} a_n z^n \)  
c. \( \sum_{n=0}^{\infty} a_n^2 z^n \).

3. (4) Let \( G_1 = \{ z : 0 < \text{Im} \, z < \pi, \text{Re} \, z > 0 \} \). Let \( f(z) = \cosh z \). Find the conformal image of \( G_1 \) under \( f \). Explicitly show that the images of the curves \( m_y = \{ z = x + iy : x > 0 \} \) and \( n_x = \{ z = x + iy : 0 < y < \pi \} \) under \( f \) intersect orthogonally in \( f(G_1) \).

4. (4) Let \( G_4 = \{ z = re^{i\theta} : 0 < r < 1, 0 < \theta < \gamma/2 \} \), i.e., \( G_4 \) is the intersection of the first quadrant with the unit disk centered at the origin (the interior thereof). Let \( G_5 = \{ z : 0 < \text{Im} \, z < 1 \} \). Find a one-to-one conformal mapping \( f \) which maps \( G_4 \) onto \( G_5 \).
5. (10) The lines, \( x = \frac{1}{2} \) and \( y = 0 \) divide \( D \), (the unit disk centered at 0) into 4 subregions \( D_1, D_2, D_3 \) and \( D_4 \). See figure to the right. Let \( w = \frac{1 + 2z}{1 - z} \). Find the images \( E_j \) of each subregion \( D_j \) under \( w \), i.e., find

\[ E_j = w(D_j), \quad j = 1, 2, 3, 4. \]

6. (4) Let \( G_2 = \{ z : \frac{1}{2} < |z| < \frac{3}{2} \} \) and let \( G_5 = \{ z = re^{i\theta} : 1 < r < \infty, 0 < \theta < \frac{\pi}{2} \} \), i.e., \( G_5 \) is the complement of the closure of the domain \( G_4 \) (referenced in Problem 4) in the first quadrant (the interior thereof). Find a one-to-one conformal mapping \( f \) which maps \( G_2 \) onto \( G_5 \).

7. (4) Let \( u(x, y) = e^{2x} \sin 2y - \cos 3x \sinh 3y - 2xy - 3y + 2 \). Show that \( u \) is harmonic on \( \mathbb{C} \) and find a harmonic conjugate \( v \) such that \( f = u + iv \) is analytic on \( \mathbb{C} \).