MATH 5320 Exam II 4 November 2013

TakeHome - Due Nov 8

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

1. (15) Discuss the convergence of <u>three</u> of the following series, i.e., determine where the series converge absolutely and where the series converge uniformly.

a. $\sum_{n=1}^{\infty} n^2 z^{n^2}$ b. $\sum_{n=1}^{\infty} \frac{1+\cos\frac{\pi}{2}n}{n} z^n$ c. $\sum_{n=1}^{\infty} \frac{1}{z^2+n^2}$

d. $\sum_{n=1}^{\infty} n^{-z}$.

2. (9) Suppose that the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ is r, where $0 < r < \infty$. Discuss the radius of convergence of each of the following series:

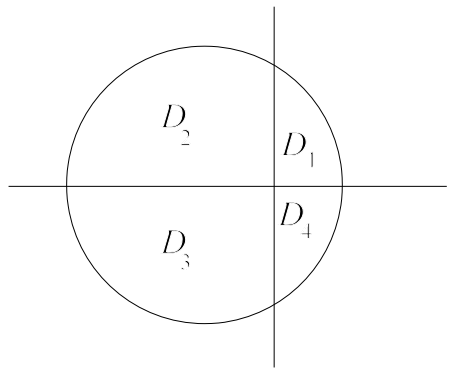
a. $\sum_{n=0}^{\infty} a_n z^{2n}$ b. $\sum_{n=0}^{\infty} a_{2n} z^n$ c. $\sum_{n=0}^{\infty} a_n^{\,2} z^n$

3. (4) Let $G_1 = \{z : 0 < \operatorname{Im} z < \pi, \operatorname{Re} z > 0\}$. Let $f(z) = \cosh z$. Find the conformal image of G_1 under f. Explicitly show that the images of the curves $m_y = \{z = x + iy : x > 0\}$ and $n_x = \{z = x + iy : 0 < y < \pi\}$ under f intersect orthogonally in $f(G_1)$.

4. (4) Let $G_4 = \{z = re^{i\theta} : 0 < r < 1, 0 < \theta < \pi/2\}$, *i.e.*, G_4 is the intersection of the first quadrant with the unit disk centered at the origin (the interior thereof). Let $G_3 = \{z : 0 < \operatorname{Im} z < 1\}$. Find a one-to-one conformal mapping f which maps G_4 onto G_3.

5. (10) The lines, $x = \frac{1}{2}$ and $y = 0$ divide D, (the unit disk centered at 0) into 4 subregions D_1, D_2, D_3 and D_4. See figure to the right. Let $w = \frac{1+2z}{1-z}$. Find the images E_j of each subregion D_j under w, i.e., find

$$E_j = w(D_j), \quad j = 1, 2, 3, 4.$$



6. (4) Let $G_2 = \{z : |z - \frac{1}{2}| < \frac{1}{2}\}$ and let $G_5 = \{z = re^{i\theta} : 1 < r < \infty, 0 < \theta < \frac{\pi}{2}\}$, *i.e.*, G_5 is the complement of the closure of the domain G_4 (referenced in Problem 4) in the first quadrant (the interior thereof). Find a one-to-one conformal mapping f which maps G_2 onto G_5.

7. (4) Let $u(x, y) = e^{2x}\sin 2y - \cos 3x \sinh 3y - 2xy - 3y + 2$. Show that u is harmonic on $\mathbb{C}$ and find a harmonic conjugate v such that $f = u + i\,v$ is analytic on $\mathbb{C}$.