Math 5320 Exam I 2 October 2013

In-Class Make-up

Due: Monday, 7 October

See instructions for corrections. Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

Part A. Do any five (5) of the following problems:

1. (12) Let $z = -4 - 4i$ *and* $w = 2 + 2\sqrt{3}\,i$. Write in rectangular form, $a + bi$, and polar form, $r\,\text{cis}\,\theta$, each of the following:

   a. $\dfrac{w^3}{z^4}$ b. $(z - \overline{w})(w - \overline{z})$

2. (12) Prove the following proposition: Let (X, d) be a metric space and let A be a subset of X. Then, $\partial A = \overline{A} \setminus \operatorname{int} A$.

3. (12) Prove the following proposition: Let (X, d) be a complete metric space and let $Y \subset X$. If Y is closed, then Y is complete.

4. (12) Prove the following proposition: Let $z, w \in \mathbb{C},\ w \neq 0$. Then, $\overline{\left(\dfrac{z}{w}\right)} = \dfrac{\overline{z}}{\overline{w}}$

5. (12) Give examples of sequences of subsets $\{F_n\}$ of $\mathbb{C}$ such that

   a. each F_n is closed and for each $n \geq 1,\ F_n \supset F_{n+1}$, but $\bigcap_{n=1}^{\infty} F_n$ is not a singleton

   b. each F_n is closed and $\lim_{n\to\infty} diam(F_n) = 0$ but $\bigcap_{n=1}^{\infty} F_n$ is not a singleton

   c. for each $n \geq 1,\ F_n \supset F_{n+1}$ and $\lim_{n\to\infty} diam(F_n) = 0$, but $\bigcap_{n=1}^{\infty} F_n$ is not a singleton

6. (12) Prove the following proposition: Let (X, d) be a metric space. If C is a component of X, then C is closed.

Part B. Do each of the following problems:

7. (15) Provide a counterexample to each of the following assertions:

a. In a metric space (X , d), for any $A \subset X$ we have $diam(\text{int}(A)) = diam(A)$

b. In a metric space (X , d), for any $A \subset X$ we have $\text{int}\,\overline{A} \subset \overline{\text{int}\,A}$

c. Let (X , d) be a metric space and let $f : X \to \mathbb{R}$ be continuous on X. If A is a closed subset of X, then $f(A)$ is a closed subset of $\mathbb{R}$.

d. Let (X , d) be a compact metric space and let $f : X \to \mathbb{R}$ be continuous on X. Then, f is Lipschitz.

e. Let (X, d) be a metric space and let $f : X \to \mathbb{R}$ be bounded and continuous on X. Then, f is uniformly continuous.

8. (30) Classification Problem. Correctly identify whether the following subsets of $\mathbb{C}$ are: (a) open; (b) closed; (c) connected; (d) polygonally path connected; (e) compact; (f) complete; (g) bounded; (h) region. You do not need to provide a rationale for your classification. Fill out the classification information on the attached page, i.e.,

if the set possesses the property **<u>mark the cell</u>** in the table with Y (= yes)
if the set does not possess the property **<u>do not mark</u>** the cell in the table

A. $\{\overline{B(0,1)} \setminus B(\tfrac{1}{4}, \tfrac{1}{4})\} \cap \{z : \text{Re}\, z > 0\}$

B. $\{B(0,1) \setminus \overline{B(\tfrac{1}{2}, \tfrac{1}{2})}\}$

C. $T \setminus B(0,1)$ where T is the equilateral triangle (interior plus the sides) centered at 0 with vertices $\left\{2, 2\text{cis}\frac{2\pi}{3}, 2\text{cis}\frac{4\pi}{3}\right\}$. Note: the circle C(0,1) is inscribed inside of T and is tangent to the side of T at the points $\{-1, \text{cis}\frac{\pi}{3}, \text{cis}\frac{-\pi}{3}\}$.

D. $\{z = x + iy : xy < 2,\ xy > 0\} \setminus \overline{B(0,4)}$

E. $\bigcup_{n=1}^{\infty} l_n$, where each $l_n = [0, b_n]$ where $b_n = \text{cis}\frac{\pi}{2n}$

Classification Table for Problem 8

	open	closed	connected	polygonally path connected	compact	complete	bounded	region
A								
B								
C								
D								
E								