MATH 5320 Exam II 3 November 2008

In-Class Make-Up
Due 7 November

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. Determine the radius of convergence of each of the following series:

a. $\sum_{n=0}^{\infty} \frac{n!n!}{(2n+1)!}(2z-i)^n$ b. $\sum_{n=1}^{\infty} \frac{z^n}{(3+(-1)^{n(n+1)/2})^n}$

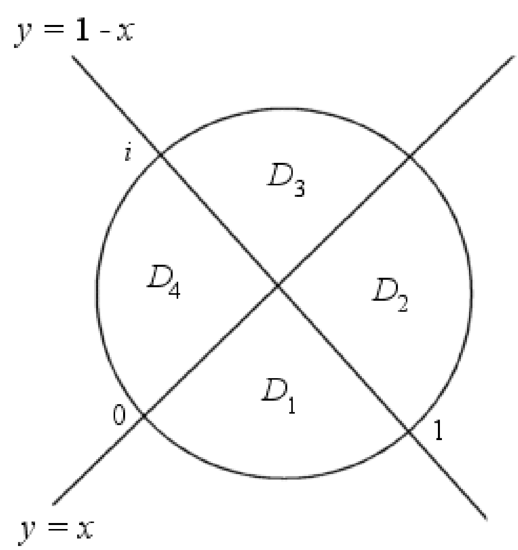
2. Let G be a region in $\mathbb{C}$ and let $f \in \mathcal{A}(G)$. Prove that if $\left(\operatorname{Re} f(z)\right)^3 + 3\operatorname{Im} f(z) = -1$ on G, then f is constant on G.

3. Show that for all complex z the following hold:

a. $\cos 2z = \cos^2 z - \sin^2 z$

b. $|\sin z|^2 = \sin^2 x + \sinh^2 y \quad \text{for} \quad z = x + iy$

4. Let $f(z) = (1-z)^{1+i}$. Identify and sketch the image of the line segment $(0, i)$ under f.



5. The circle C, which passes through the three points i, 0, 1, is divided into four congruent regions by the lines

$y = x$ and $y = 1 - x$

(say, D_1, D_2, D_3 and D_4). See figure to the right. Let $w = \frac{z-1}{z-i}$. Find/identify the images E_j of each subregion D_j under w, i.e., find $E_j = w(D_j), \quad j = 1, 2, 3, 4.$