1. Determine the radius of convergence of each of the following series:
   a. \[ \sum_{n=0}^{\infty} \frac{n!n!}{(2n+1)!} (2z-i)^n \]
   b. \[ \sum_{n=1}^{\infty} \frac{z^n}{(3 + (-1)^{n(n+1)/2})^n} \]

2. Let \( G \) be a region in \( \mathbb{C} \) and let \( f \in \mathcal{A}(G) \). Prove that if \((\text{Re } f(z))^3 + 3 \text{Im } f(z) = -1\) on \( G \), then \( f \) is constant on \( G \).

3. Show that for all complex \( z \) the following hold:
   a. \( \cos 2z = \cos^2 z - \sin^2 z \)
   b. \( |\sin z|^2 = \sin^2 x + \sinh^2 y \) for \( z = x + iy \)

4. Let \( f(z) = (1 - z)^{1+i} \). Identify and sketch the image of the line segment \((0, i)\) under \( f \).

5. The circle \( C \), which passes through the three points \( i, 0, 1 \), is divided into four congruent regions by the lines
   \( y = x \) and \( y = 1 - x \)
   (say, \( D_1, D_2, D_3 \) and \( D_4 \)). See figure to the right. Let \( w = \frac{z-1}{z-i} \). Find/identify the images \( E_j \) of each subregion \( D_j \) under \( w \), i.e., find \( E_j = w(D_j) \), \( j = 1, 2, 3, 4 \).