1. (15) Discuss the convergence of five of the following series, i.e., determine where the series converge, where the series converge absolutely and where the series converge uniformly.

   a. \( \sum_{n=0}^{\infty} \left( \frac{z}{z+1} \right)^n \)

   b. \( \sum_{n=1}^{\infty} n^2 z^n \)

   c. \( \sum_{n=1}^{\infty} \frac{z^n}{1-z^n} \)

   d. \( \sum_{n=1}^{\infty} \frac{1}{z^2 + n^2} \)

   e. \( \sum_{n=1}^{\infty} n^{-z} \)

   f. \( \sum_{n=0}^{\infty} e^{-nz^2} \)

2. (9) Suppose that the radius of convergence of \( \sum_{n=0}^{\infty} a_n z^n \) is \( r \), where \( 0 < r < \infty \). Find the radius of convergence of each of the following series:

   a. \( \sum_{n=0}^{\infty} a_n z^{2n} \)

   b. \( \sum_{n=0}^{\infty} a_{2n} z^n \)

   c. \( \sum_{n=0}^{\infty} a_n^2 z^n \)

3. (4) Let \( G_1 = \{ z : \text{Im} z < \frac{\pi}{2}, \text{Re} z > 0 \} \). Let \( f(z) = \sinh z \). Find the conformal image of \( G_1 \) under \( f \). Explicitly show that the images of the curves \( m_y = \{ z = x + iy : x > 0 \} \) and \( n_x = \{ z = x + iy : -\frac{\pi}{2} < y < \frac{\pi}{2} \} \) under \( f \) intersect orthogonally in \( f(G_1) \).

4. (4) Let \( G_2 = \{ z : |z - \frac{1}{2}| < \frac{1}{2} \} \) and \( G_3 = \{ z : 0 < \text{Im} z < 2 \} \). Find a one-to-one conformal mapping \( f \) which maps \( G_2 \) onto \( G_3 \).
5. (10) The lines, $x = \frac{1}{2}$ and $y = 0$ divide $D$, (the unit disk centered at 0) into 4 subregions $D_1, D_2, D_3$ and $D_4$.

See figure to the right. Let $w = \frac{z}{z-1}$. Find the images $E_j$ of each subregion $D_j$ under $w$, i.e., find $E_j = w(D_j), \ j = 1, 2, 3, 4$.

6. (4) Let $G_4 = \{ z = re^{i\theta} : 0 < r < 1, 0 < \theta < \frac{\pi}{2} \}$ and let $G_5 = \{ z = re^{i\theta} : 1 < r < \infty, 0 < \theta < \frac{\pi}{2} \}$, i.e., $G_4$ is the intersection of the first quadrant with the unit disk centered at the origin (the interior thereof) and $G_5$ is the complement of the closure of $G_4$ in the first quadrant (the interior thereof). Find a one-to-one conformal mapping $f$ which maps $G_4$ onto $G_5$.

7. (4) Let $u(x, y) = x^3 - 3xy^2 + 2e^y \sin x - 2y + 5$. Show that $u$ is harmonic on $RHP$ (the open right half-plane) and find a harmonic conjugate $v$ such that $f = u + iv$ is analytic on $RHP$.