MATH 5320 Exam II 31 October 2008

In-Class

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **<u>Show all relevant supporting steps!</u>**

1. Determine the radius of convergence of each of the following series:

 a. $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+4}{n-1}(2-z)^n$ b. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{4+(-1)^n}{9+(-1)^{n+1}}\right)^n z^n$

2. Let G be a region in $\mathbb{C}$ and let $f \in \mathcal{A}(G)$. Prove that if $\operatorname{Re} f(z) + (\operatorname{Im} f(z))^2 = 5$ on G, then f is constant on G.

3. Show that for all complex z the following hold:

 a. $\sinh(z+w) = \sinh z \cosh w + \cosh z \sinh w$

 b. $|\cos z|^2 = \cos^2 x + \sinh^2 y$ for $z = x + iy$

4. Let $f(z) = z^i$. Identify and sketch the image of the line segment (0, i) under f.

5. The circle C, which passes through the three points i, 0, 1, is divided into four congruent regions by the lines

 $y = x$ and $y = 1 - x$

 (say, D_1, D_2, D_3 and D_4). See figure to the right. Let $w = \dfrac{z-1-i}{z}$. Find/identify the images E_j of each subregion D_j under w, i.e., find $E_j = w(D_j)$, $j = 1, 2, 3, 4$.