MATH 5320 Exam II November 5, 2004

In-Class Make-Up

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. Determine the radius of convergence of each of the following series:

   a. $$\sum_{n=2}^{\infty}\left(\frac{n+(-1)^n}{n^2+(-1)^n}\right)^n (2z-i)^n$$

   b. $$\sum_{n=2}^{\infty}\left(\frac{4+(-1)^n}{9+(-1)^{n+1}}\right)^n (z+1)^n$$

2. Let G be a region in $\mathbb{C}$ and let $f \in A(G)$. Prove that if $\operatorname{Re} f(z) + \operatorname{Im} f(z) \equiv 0$ on G, then f is constant on G.

3. Show that for all complex z the following hold:

   a. $|\cos z|^2 = \cos^2 x + \sinh^2 y \quad \text{for} \quad z = x + iy$

   b. $\cos 3z = \cos^3 z - 3\cos z \sin^2 z$

4. Let $f(z) = (1-z)^{1+i}$. Identify and sketch the image of the line segment $(0, i)$ under f.

5. Let M be the Möbius transformation which maps $i - 1, 2i, i + 1$ to $\frac{1}{i+1}, \frac{1}{2i}, \frac{1}{i-1}$, resp. Find a formula for M and identify images of the unit quarter discs under M, i.e., the images of D_1, D_2, D_3, D_4, where the unit quarter disc D_j is given by $D_j = Q_j \cap B(0,1)$. See figure.