

## In-Class

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. Determine the radius of convergence of each of the following series:

a. 
$$\sum_{n=1}^{\infty} n^2 (2z - i)^n$$

b. 
$$\sum_{n=1}^{\infty} \frac{z^n}{(3 + i^n)^n}$$

2. Let  $G$  be a region in  $\mathbb{C}$  and let  $f : G \rightarrow \mathbb{C}$ . Prove that if both  $f$  and  $\overline{f}$  are analytic on  $G$ , then  $f$  is constant on  $G$ .

3. Show that for all complex  $z$  the following hold:

a. 
$$\cosh^2 z - \sinh^2 z = 1$$

b. 
$$\cos 2z = \cos^2 z - \sin^2 z$$

4. Let  $f(z) = z^{1-i}$ . Identify and sketch the image of the line segment  $(0, i)$  under  $f$ .

5. Let  $M$  be the Möbius transformation which maps  $1+i$ ,

$0$ ,  $1-i$  to  $\frac{i}{i-1}$ ,  $\frac{1}{2}$ ,  $\frac{i}{i+1}$ , resp. Find a formula

for  $M$  and identify images of the unit quarter discs under  $M$ , i.e., the images of  $D_1, D_2, D_3, D_4$ ,

where the unit quarter disc  $D_j$  is given by

$D_j = Q_j \cap B(0,1)$ . See figure.

