MATH 5320 Exam I September 27, 2004

TakeHome - Due October 4

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (6) For $z \in \mathbb{C}$ let Z denote the point on S which is the stereographic projection of z. For $z \in \mathbb{C} \setminus \{0\}$ let z^* denote the point in $\mathbb{C}$ such that $Z^* = -Z$. Prove or disprove: If T, the triangle determined by the vertices $\{z_1, z_2, z_3\}$, for which none of the vertices $z_j = 0$, $j = 1, 2, 3$, is a right triangle, then T^*, the triangle determined by the vertices $\{z_1^*, z_2^*, z_3^*\}$, is a right triangle.

2. (6) Let $S = \{z : |z - ½| \le ½\}$. Let $T = \{w : w = \dfrac{1}{1-z}, z \in S \setminus \{1\}\}$. Geometrically describe the set T.

3. (8) For each of the following cases, give an example of a metric space (X, d), where $X \subset \mathbb{C}$ and d is the inherited metric such that

   a. there exists a sequence $\{z_n\} \subset X$ such that no subsequence of $\{z_n\}$ converges.

   b. there exists a sequence $\{z_n\} \subset X$ such that $\{z_n\}$ is Cauchy, but $\{z_n\}$ does not converge.

   c. there exists a sequence $\{z_n\} \subset X$ such that there exists two subsequences of $\{z_n\}$, say $\{z_{n_j}\}$ and $\{z_{n_k}\}$, such that $\{z_{n_j}\} \to z'$ and $\{z_{n_k}\} \to z''$ and $z' \ne z''$.

   d. every point of X is a limit point of X.

4. Omit.

5. (6) Let $M = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in \mathbb{R} \right\}$ and let $\oplus$, $\odot$ denote matrix addition and matrix multiplication, resp.. Show that $(\mathbb{C}, +, \bullet)$ and $(M, \oplus, \odot)$ are isomorphic (fields).

6. (12) Sketch the set:

a. $|z| \arg z = \dfrac{p}{2}$

b. $|z+1| = |2z-1|$

In 7, for metric spaces (X, d) and (Ω, r) let $Hom(X, Y)$ denote the set of homeomorphisms between X and Y, i.e., the set of all functions $f : X \to Y$ such that: (i) f is continuous on X ; (ii) f is one-to-one; (iii) f is onto; (iv) f^{-1} is continuous on Y.

7. (6) Let $f : \mathbb{C} \to \mathbb{C}$ be given by $f(z) = \dfrac{z}{1+|z|}$. Show that $f \in Hom(\mathbb{C}, B(0,1))$.

8. (6) Let (X, d) be a metric space and let A, B be subset of X such that $A \subset B$. Show that $\overline{A} \subset \overline{B}$.

Extra Credit. (3)

For metric spaces (X, d) and (Ω, r), give an example which shows that there exists $f : X \to Y$, f is one-to-one on X, f is onto Y, but $f \notin Hom(X, Y)$.