MATH 5320 Exam II November 17, 1997

Notation. D denotes the open unit disk centered at 0. *RHP* denotes the open right-half plane. Q_1 denotes the open first quadrant.

Part A. In class.

1. Prove: There does not exist a branch-of-log on $B(0,1)\backslash\{0\}$.

2. Let G be a region in $\mathbb{C}$ and let $f \in \mathcal{A}(G)$, $f = u + iv$. Show that if there exist constants α, β and γ (not all 0) such $\alpha u + \beta v = \gamma$ on G, then f is necessarily constant.

3. Show that for all complex z and w the following holds:
   (a) $\cosh(z+w) = \cosh(z)\cosh(w) + \sinh(z)\sinh(w)$
   (b) $\cosh^2(z) + \sinh^2(w) = \sinh^2(z) + \cosh^2(w)$

4. Let $D_1 = Q_1 \cap D$. Determine (describe, identify) the image of D_1 under the conformal mapping $f(z) = z^i$.

5. Find the bilinear transformation which maps the points -1, 0, 1 to the points 0, ∞, i, respectively. Describe the image of the exterior of the union of the disks B(-1,1) and B(1,1).