Math 5318 Exam II November 6, 2000

In Class Portion:
All outside references must be to definitions and/or theorems in Chapters 1.1 - 3.6 & Appendix.

1. Prove: Theorem 3.1B (part II): *If* $\sum_{n=1}^{\infty} a_n$ converges to $A$ and if $c \in \mathbb{R}$, then $\sum_{n=1}^{\infty} c\, a_n$ converges to $cA$.

2. Find: a) $\lim_{n\to\infty} \dfrac{\sin\left(\dfrac{n^4+1}{n^2+1}\right)}{n}$ b) $\lim_{n\to\infty} \dfrac{\sqrt{2n^2+1}}{2n-1}$

3. Suppose $\{s_n\}_{n=1}^{\infty}$ is a bounded sequence and $\liminf_{n\to\infty} s_n = m$. Prove there exists a subsequence $\{s_{n_k}\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} s_{n_k} = m$.

4. Give an example which shows that the conclusion of the Nested Interval Theorem may fail if the hypothesis (in the theorem) that the intervals are nested is dropped.

5. Show that if the sequence $\{s_n\}_{n=1}^{\infty}$ is monotone non-decreasing, then for $\sigma_n = \dfrac{s_1 + s_2 + \cdots + s_n}{n}$ the sequence $\{\sigma_n\}_{n=1}^{\infty}$ is monotone non-decreasing.

6. Determine whether the following series converge:

a) $\sum_{k=1}^{\infty} \left(\dfrac{k+1}{2k+3}\right)^k$ b) $\sum_{k=1}^{\infty} \left(\dfrac{2+(-1)^k}{3}\right)^k$ c) $\sum_{k=2}^{\infty} \dfrac{2k+3}{k^3-2k+1}$

7. Find all real values $x$ for which the series $\sum_{k=1}^{\infty} \dfrac{(-1)^k x^k}{\sqrt{k^2+2}}$ converges.

Take Home Portion (Due Friday, Noon):
All outside references must be to definitions and/or theorems in Chapters 1.1 - 3.6 & Appendix.

1. Find $\lim_{n\to\infty} x_n$

   a) where $\begin{cases} x_1 \geq 0 \\ x_{n+1} = \sqrt{x_n + 2}, \ n > 0 \end{cases}$

   b) where $\begin{cases} 0 \leq x_1 \leq 1 \\ x_{n+1} = 1 - \sqrt{1 - x_n}, \ n > 0 \end{cases}$

2. Give examples of sequences $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ such that $s_n \to 0$ and $t_n \to \infty$ and

   a) $s_n t_n \to 0$ b) $s_n t_n \to 3$ c) $s_n t_n \to -3$ d) $s_n t_n \to \infty$

3. Determine whether the following series converge:

   a) $\sum_{k=1}^{\infty} \cos \frac{1}{k^2}$

   b) $\sum_{k=1}^{\infty} \sin \frac{1}{k^2}$

   c) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$

4. Find all real values of $p$ such that the series $\sum_{k=2}^{\infty} \frac{1}{k \log^p k}$ converges.

5. Find all real values of $x$ for which the series $\sum_{k=1}^{\infty} (3 + (-1)^k)(x-1)^k$ converges.

6. Determine whether the following series converge absolute, converge conditionally or diverge:

   a) $\sum_{k=1}^{\infty} \frac{(-1)(-3)\cdots(1-2k)}{1\cdot 4\cdots(3k-2)}$

   b) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sqrt{k}}{k+1}$

7. Estimate the value of the series $\sum_{k=1}^{\infty} \log\left(1 + \frac{(-1)^{k+1}}{k^3}\right)$ to within an accuracy of 0.001. Justify your estimate.