Review for Final Exam

Using a graphing calculator or a CAS (either WolframAlpha or Maple):

1.   Consider the following polynomials

$$f(x) = 2x^2 + 3x - 21$$
$$g(x) = 12x^3 - 28x^2 + 3x + 18$$
$$h(x) = 2x^4 - 9x^3 + 15x^2 - 36$$

Construct answers for each of the polynomials f, g and h to the following queries:

A. How many roots does the polynomial have?
B. How many real roots does the polynomial have?
C. How many complex roots does the polynomial have?
C. How many x-intercepts does the polynomial have?
D. Factor the polynomial completely into irreducible polynomials over the reals.
E. Copy a graph of the polynomial which clearly shows the behavior of the polynomial near the roots from a CAS into the worksheet.

2.   Consider the following sets:

$\alpha = \{-2,-2,5\}$
$\beta = \{-4, -2+i, -2-I\}$
$\gamma = \{-4, -3, -1, 2, 2, 5\}$

Construct answers for each of the sets α, β and γ to the following queries:

A. Find a polynomial who roots are exactly the elements of the set. (Write the polynomial out in expanded form.)
B. Factor the polynomial given in A into irreducible polynomials over the reals. Copy a graph of the polynomial given A which clearly shows the behavior of the polynomial near the roots from a CAS into the worksheet.
How many x-intercepts does the polynomial given A have?

3.   Consider the following two sets of polynomials:

α $\quad f(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8,$

$\quad\quad g(x) = x^7 - 14x^6 + 78x^5 - 224x^4 + 361x^3 - 330x^2 + 160x - 32,$

β $\quad F(x) = x^5 - 6x^4 + 14x^3 - 16x^2 + 9x - 2,$

$\quad\quad G(x) = x^7 - 16x^6 + 105x^5 - 366x^4 + 732x^3 - 840x^2 + 512x - 128$

Construct answers for each of the sets α and β to the following queries:

A. Find the greatest common divisor of the two polynomials.
B. Find the least common multiple of the two polynomials.

4.   Consider the functions

$$y_{-2} = 2x^2 - 2x - 1$$

$$y_{-1} = 2x^2 - x$$

$$y_0 = 2x^2 + 1$$

$$y_1 = 2x^2 + x + 2$$

$$y_2 = 2x^2 + 2x + 3$$

A.   Plot the graphs all five functions on the same coordinate axes.
B.   Which of the five functions have only complex roots?
C.   Estimate the minimum value of each function.
D.   Estimate the x-intercepts of those functions for which real roots exist.

5.   Consider the functions

$$f(x) = \frac{2x-1}{x^2-x-6},$$

$$g(x) = \frac{x^2-x-6}{2x-1},$$

$$h(x) = \frac{20+x^2}{2x^2+3x+12}$$

Construct answers for each of the functions *f, g* and *h* to the following queries:

A. Does the function have vertical asymptotes? If so, where?
B. Does the function have a horizontal asymptote? If so, where?
C. Does the function have a slant asymptote?
D. Copy a graph of the function which clearly shows the behavior of the function near any intercepts (if any) and also shows the asymptotes (if any) from a CAS into the worksheet.

6.   Using a CAS construct an implicit plot of the equation $x^2 - 2y^2 = 8$
Copy a graph of the equation from a CAS into the worksheet.

7.   Using a CAS construct an plot of the 3-d graph of the function $f(x, y) = \dfrac{\cos(xy)}{x^2 + 2y^2 + 1}$

Copy a graph of the function from a CAS into the worksheet.

8.   For each of the following sequences identify the next two missing terms and describe the pattern that the terms in the sequence follow.

{1, 3, 6, 10, _____, _____, ...}

Pattern: _____

{1, 2, 3, 6, 10, _____, _____, ...}

Pattern: _____

9. For each of the following sequences find the specified missing terms. S is an arithmetic sequence for which the 6th and 10th terms are 11 and 95, resp.
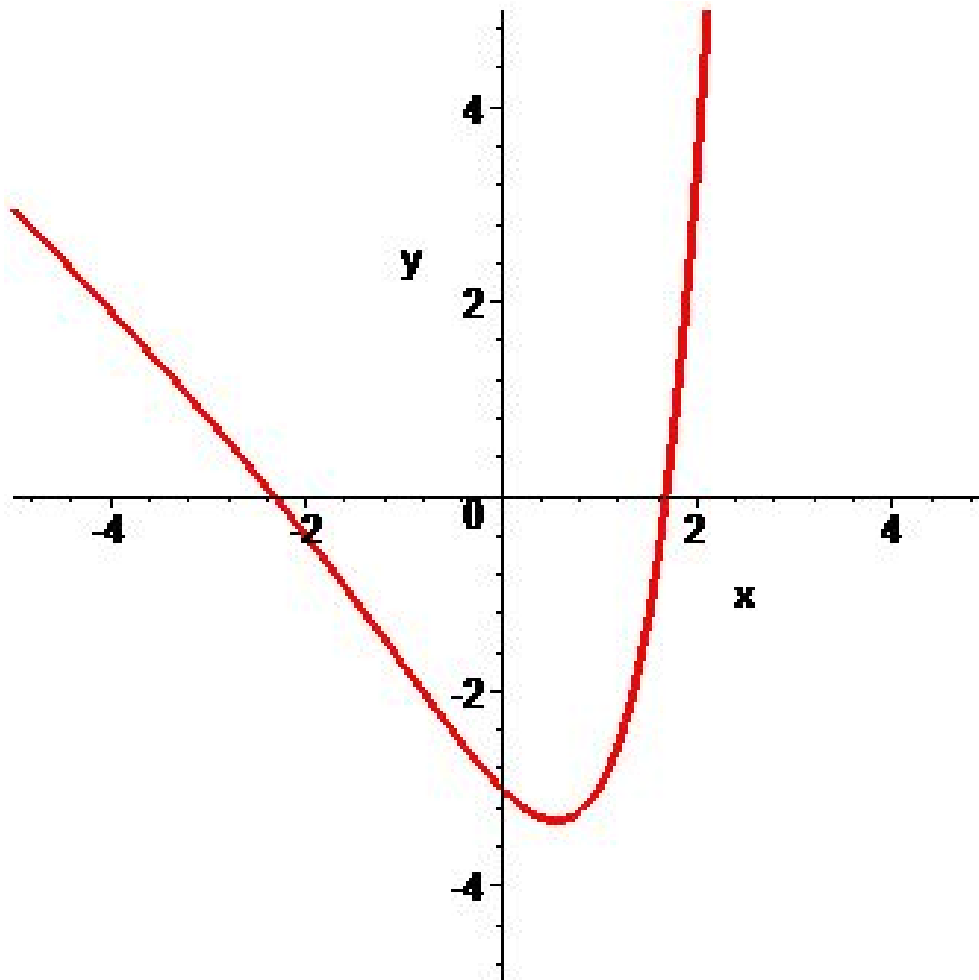
Find the 1st term and the 27th term: $_1a$ _____ $a = $, $a = $ _____

10. For each of the following sequences find an explicit functional formula for generating all of the terms of the sequence and find the 39th term of the sequence.

$\{3, 5, 11, 21, 35, \ldots\}$ where the second differences are constant.

Formula: _____ $_{39}a = $ _____

11. Sketch on the same graph the derivative and the antiderivative of function pictured below.

12.   In the picture below the graphs of three functions are plotted on the same coordinate axes.
Of the three graphs one is the graph of a function f, one is the graph of the derivative of f
and one is the graph of an antiderivative of f. Identify which graph is which:
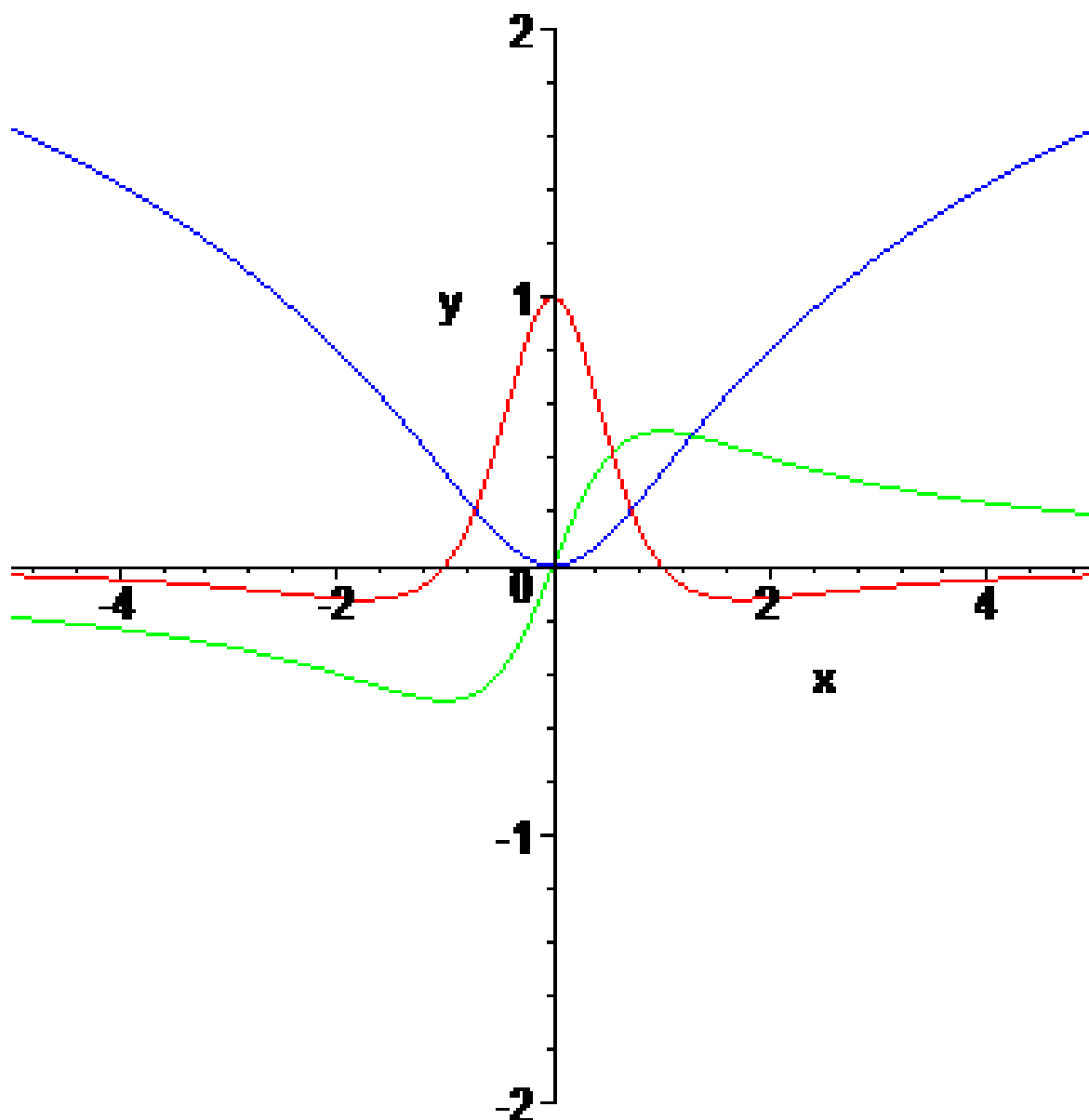
Function = _____   Derivative = _____   Antiderivative = _____