MATH 4356-001 Exam II October 24, 2003

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

Part I. Do three (3) of the following:

1. Using the definition of the derivative, determine where $x^2 + iy^2$ is differentiable.

2. Find the derivative of $w = \dfrac{(z+2)^2}{z^2 + 2i}$ (simplify).

3. Where is $f(z) = \dfrac{iz^2 + 2z}{z^3 + 2z}$ not analytic?

4. Determine whether the following statements are True or False. (Respond only with T or F)

   a. if $f(z)$ is entire, then $f(z^3)$ is entire

   b. if $f(z)$ and $g(z)$ are entire, then $f(g(z))$ is entire

   c. if $f(z)$ is entire, then $f(1/z)$ is entire

   d. if $f(z)$ and $g(z)$ are entire, then $\dfrac{f(z)}{g^2(z)+1}$ is entire

---

Part II. Do two (2) of the following:

5. Determine where $f(z) = \cos x \sinh y - i \sin x \cosh y$ is analytic. (Here $z = x + iy$.)

6. Let $f(z) = \cos 2z - 2\cos^2 z$. Show that f is constant.

7. Let $u = 3x - y - 2xy$. Show that u is harmonic on $\mathbb{C}$ and find a harmonic conjugate for u on $\mathbb{C}$.

Part IV. Do one (1) of the following:

9. Let $p(z) = z^4 - 8z^2 + 9z - 2$. Write *p* in factored form (as the product of linear factors).

10. Let $r(z) = \dfrac{z^2 - 3z - 1}{(z+1)^2(z-2)}$. Find a partial fraction expansion for *r*.

Part V. Do four (4) of the following:

11. Write $\sin(2i - 4)$ in $a + bi$ form.

12. Find the derivative of $w = \dfrac{\sinh z + \cosh z}{\sin z + \cos z}$

13. Find all solutions of $e^{4z} = i$.

14. Determine the domain of analyticity of $\mathrm{Log}(z - i) + \mathrm{Log}(z + i)$.

15. Determine the domain of analyticity of $\mathrm{Log}(z^2 + 1)$.

16. Find all values of 2^{p-i}.

17. Find the principal value of $(1+i)^{1-i}$.

Part VI. Do four (4) of the following:

18. Find an admissible parametrization for the arc of the circle $|z - 3i| = 4$ which starts at $3i + 4$ and travels in a clockwise direction to $-i$.

19. Find the length of the contour Γ given by the parametrization $z(t) = 3e^{(1-i)t}$, $-p \le t \le p$.

20. Evaluate the integral $\displaystyle\int_{-2}^{0} (1+i)\sin(it)\,dt$

21. Evaluate the contour integral $\displaystyle\int_{\Gamma} \bar{z}\,dz$ where Γ is the contour that consists of the straight line segment from 0 to $1+ i$ followed by the straight line segment from $1+ i$ to 2.

22. Evaluate the contour integral $\displaystyle\int_{\Gamma} \bar{z}^{-2}\,dz$ where Γ is the contour that consists of the arc of the unit circle which

goes from 1 to -1 in the counterclockwise direction.