MATH 4356-001 Exam I September 25, 2003

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

Notation: $Q_1, Q_2, Q_3, Q_4$ will denote the four standard quadrants.
DOOTF will stand for *Do One Of The Following*

1. DOOTF: Write each number in $a + bi$ form:

   a. $\dfrac{i}{1+i} - \dfrac{1}{i-1}$ b. $(2+i)^2(-3+4i)$

2. DOOTF: Solve for $z$ :

   a. $\dfrac{z}{1-z} = 3-4i$ b. $\dfrac{z+2}{z} = 2i-3$

3. DOOTF: Determine which quadrant(s) $\dfrac{1}{z}$ and $-\dfrac{1}{z}$ belong to if :

   a. $z \in Q_3$ b. $z \in Q_4$

4. Show that if $|z|=1, z \neq 1$, then $\operatorname{Re}\dfrac{1+z}{1-z} = 0$.

5. DOOTF: Describe the set of points $z \in \mathbb{C}$ which satisfy:

   a. $|z-1-4i| = 6$ b. $|z-3i| > 2$

6. Find the argument of the following number and write it in polar form $re^{iq}$ :

   $(1+i)(\sqrt{3}-i)$

7. Sketch the curve: $z(t) = e^{(i-1)t}$, $0 \le t \le p$

8. DOOTF: Find (all) value(s) of :

   a. $\left(2-2\sqrt{3}\,i\right)^{11}$ b. $(i-1)^{1/3}$

9. Let $A = \{z : |z-3+i| = 2\}$ $B = \{z : |z-i-1| > 1\}$

$C = \{z : 0 < |z-3i| \le 1\}$ $D = \{z : (\mathrm{Re}\, z)^2 > 0\}$

a. Which of the above sets are open?
b. Which of the above sets are domains?
c. Which of the above sets are bounded?

10. Suppose $u(x, y)$ is a real-valued function defined on a domain *D*. If

$$\frac{\partial u}{\partial x} = x \text{ and } \frac{\partial u}{\partial y} = -2y \text{ for all points in D,}$$

then show that $u(x,y) = \frac{x^2}{2} - y^2 + c$ for some constant *c*.

11. DOOTF: Describe the range of :

a. $f(z) = z^2$ for $z \in Q_2$ b. $g(z) = \frac{1}{z}$ for $|z| > 2$

12. DOOTF: Let $F(z) = z = 3i$, $G(z) = e^{ip/3}z$, $H(z) = 2z$. Consider the set $S = \{z : |z| < 1 \text{ and } \mathrm{Re}\, z > 0\}$. Sketch the image of *S* under:

a. $F(H(z))$ b. $G(F(z))$

13. DOOTF: Decide whether the sequence converges; if so, find its limit:

a. $z_n = e^{i\left(1+\frac{i}{n}\right)}$ b. $z_n = \left(\frac{1-i}{2}\right)^n$

14. DOOTF: Find the limit, if it exists:

a. $\lim_{z \to 2-3i} \left(z^2 + 6z\right)^2$ b. $\lim_{z \to i-1} \frac{z^2 - (3+i)z}{iz}$

15. DOOTF: Find the limit, if it exists:

a. $\lim_{z \to p\, i} \frac{e^z + e^{-z}}{2i}$ b. $\lim_{z \to -2i} \frac{z^2 + 8}{2z^2 + 8}$