

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

1. [12 pts.] Write each of the following complex numbers z in the form $a + bi$:

a. $z = \frac{-1 + 5i}{2 + 3i}$

b. $z = 2e^{3+ix/6}$

2. [6 pts.] Write the complex number $\frac{2 + 2i}{-\sqrt{3} + i}$ in the form $re^{i\theta}$.

3. [16 pts.] Sketch the graphs of:

a. $z(t) = -1 + i + t(2 - i), 0 \leq t \leq 1$ b. $z(t) = e^{-1+it}, 0 < t < 1$

4. [6 pts.] Solve $z^2 + 2iz - 5 = 0$.

5. [8 pts.] Show that $\text{Im } z > 0$ implies that $\text{Im } 1/z < 0$.

6. [8 pts.] Find all values of $(1 - \sqrt{3}i)^{\frac{1}{3}}$

7. [10 pts.] Let $S = \{ z : \text{Re } z > 0 \text{ and } 1 < |z| < 2 \}$ and let $T = \{ z : |z| < 1 \} \setminus (0,1)$. For each of the sets S and T determine whether the set is:

a. open b. closed c. connected d. convex e. bounded

8. [12 pts.] Find each of the following limits (if they exist):

a. $\lim_{z \rightarrow \infty} \left(\frac{1-i}{4} \right)^z$

b. $\lim_{z \rightarrow -i} \frac{z^2 + 1}{z^4 - 1}$

9. [8 pts.] Determine where $f(x,y) = x + \frac{y}{x^2 + y^2} + i(y + \frac{x}{x^2 + y^2})$ is analytic (if anywhere).

10. [9 pts.] Show that $y^2 - x^2 - y$ is harmonic and find a harmonic conjugate for it.

11. [6 pts.] Give an example for each of the following:

a. A function f which is defined on an open set D such that f is analytic on D and $\text{Re } f(z) \equiv 0$ on D , but f is not constant on D .

b. A function f which is defined on an domain D such that $\text{Re } f(z) \equiv 0$ on D , but f is

not constant on D .