MATH 3360-001 Exam II 15 October 2010

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. (12 pts) Complete each of the following definitions:

   a. A group G is called ***cyclic*** if and only if . . .
   b. The ***symmetric group,*** S_n ***, of degree n*** is . . .
   c. Let G be a group and let H be a subgroup of G. A ***left coset*** *of H in G* is . . .

2. (8 pts) Given an example of a group with the indicated combination of properties

   a. finite cyclic group with exactly 8 generators
   b. infinite Abelian group that is not cyclic
   c. finite Abelian group that is not cyclic
   d. finite non-abelian group of order more than 6

3. (14 pts) Let $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 8 & 2 & 4 & 1 & 7 & 6 \end{pmatrix}$

   and let $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 1 & 5 & 7 & 3 & 6 & 2 & 4 \end{pmatrix}$

   a. (4 pts) Calculate $\phi\tau$ and $\tau\phi$
   b. (4 pts) Calculate $\phi^2\tau$
   c. (6 pts) Write ϕ and τ as products of disjoint cycles and calculate $|\phi|$ and $|\tau|$

4. (12 pts) Determine which of the follow subsets of S_7 are subgroups of S_7 ?

   a. $H_1 = \{(1\,2\,5), (2\,5), (1\,5\,2), (1)\}$
   b. $H_2 = \{(1\,4), (2\,6\,7), (2\,7\,6), (1\,4)(2\,6\,7), (1\,4)(2\,7\,6), (1)\}$
   c. $H_3 = \{(1\,4), (1\,6\,7), (1\,7\,6), (1\,4)(1\,6\,7), (1\,4)(1\,7\,6), (1)\}$
   d. $H_4 = \{(1\,4), (3\,7), (1\,4)(3\,7), (1)\}$

5. (12 pts) Identify the cosets of $< 6 >$ in $\mathbb{Z}_{18}$.

6. (12 pts) Let $H = \{\phi \in S_6 : \phi(1) = 1 \text{ and } \phi(6) = 6\}$. Find the index of *H* in S_6.

7. (14 pts) Let *H* be a subgroup of *G*. For $a, b \in G$, let $a \sim b$ if and only if $ab^{-1} \in H$. Prove that ~ is an equivalence relation on *G*.

8. (8 pts) Let *G* be a group such that $|G| < 500$. Suppose that *H* is a subgroup of *G* such that $|H| = 48$ and *K* is a subgroup of *G* such that $|K| = 54$. What is the order of *G* ?

9.(12 pts) Find the remainder of 19^{1763} upon division by 23.