1. (12 pts) Complete each of the following statements:
   a. Let $\alpha : A \rightarrow B$. $\alpha$ is one-to-one if and only if . . . .
   b. Let $\alpha : A \rightarrow B$. $\alpha$ is invertible if and only if . . . .
   c. Let $(G, \ast)$ be a group. A non-empty subset $H$ of $G$ is a subgroup of $G$ if and only if . . . .

2. (12 pts) Consider the Euclidean plane $\mathbb{R}^2$. Determine whether the relation $\sim$ defined on $\mathbb{R}^2$ by $(p_1, p_2) \sim (q_1, q_2)$ means $p_1 = q_1$ or $p_2 = q_2$ is an equivalence relation.

3. (12 pts) Use the Euclidean algorithm to find the greatest common divisor of 192 and 118.

4. (12 pts) Calculate the given expression and express the result in rectangular form:
   a. $\frac{2 - 3i}{1 + i}$
   b. $(1 + i)^{13}$

5. (12 pts) Calculate the following matrix computations
   a. $\begin{bmatrix} 6 & 4 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ in $M(2, \mathbb{Z}_{12})$
   b. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^7$ in $M(2, \mathbb{Z}_7)$

6. (24 pts) Consider the following sets $G$ with associative operations $\ast$. In each case, determine whether $(G, \ast)$ is a group. If $(G, \ast)$ is not a group, state why it is not.
   a. $G = \{0, 1, 2, 3, 4, \ldots \}$, $\ast$ is addition of integers
   b. $G = \mathbb{Q} \setminus \{0\}$, i.e., $G$ is the set of non-zero rational numbers, $\ast$ is multiplication of rational numbers
   c. $G = M(2, \mathbb{Q})^\ast$, i.e., $G$ is the set of 2x2 non-zero matrices with rational entries, $\ast$ is matrix multiplication
7. (12 pts) Complete the following table so that \( G = \{a, b, c, d\} \) with \( * \) is a commutative group:

<table>
<thead>
<tr>
<th>*</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<tbody>
<tr>
<td>a</td>
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<tr>
<td>b</td>
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<td>c</td>
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<tr>
<td>d</td>
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</tbody>
</table>

8. (12 pts) Determine whether the following sets \( H \) are subgroups of the given groups \( G \) with operations \( * \):

   a. \( H = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\} \), i.e., \( H \) is the set of real numbers of the form \( a + b\sqrt{2} \) where \( a \) and \( b \) are integers not both zero; \( G = \mathbb{R}^* \), i.e., \( \mathbb{R}^* \) is the set on non-zero real numbers; \( * \) is multiplication of real numbers.

   b. \( H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in 3\mathbb{Z} \right\} ; \ G = M(2, \mathbb{Z}) ; * \) is matrix addition.