MATH 3360-001 Exam I 24 September 2010

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. (12 pts) Complete each of the following statements:

   a. Let $\alpha : A \to B$. α is one-to-one if and only if . . . .

   b. Let $\alpha : A \to B$. α is invertible if and only if . . . .

   c. Let (G,*) be a group. A non-empty subset H of G is a subgroup of G if and only if . . . .

2. (12 pts) Consider the Eucidean plane $\mathbb{R}^2$. Determine whether the relation ~ defined on $\mathbb{R}^2$ by $(p_1, p_2) \sim (q_1, q_2)$ means $p_1 = q_1$ or $p_2 = q_2$ is an equivalence relation.

3. (12 pts) Use the Euclidean algorithm to find the greatest common divisor of 192 and 118.

4. (12 pts) Calculate the given expression and express the result in rectangular form:

   a. $\dfrac{2-3i}{1+i}$ b. $(1+i)^{13}$

5. (12 pts) Calculate the following matrix computations

   a. $\begin{bmatrix} 6 & 4 \\ 3 & 10 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 7 & 1 \end{bmatrix}$ in $M(2, \mathbb{Z}_{12})$ b. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}^7$ in $M(2, \mathbb{Z}_8)$

6. (24 pts) Consider the following sets G with associative operations *. In each case, determine whether (G, *) is a group. If (G,*) is not a group, state why it is not.

   a. $G = \{0, 1, 2, 3, 4, . . \}$, * is addition of integers

   b. $G = \mathbb{Q} \setminus \{0\}$, i.e., G is the set of non-zero rational numbers, * is multiplication of rational numbers

   c. $G = M(2, \mathbb{Q})^*$, i.e., G is the set of 2×2 non-zero matrices with rational entries, * is matrix multiplication

7. (12 pts) Complete the following table so that $G = \{a, b, c, d\}$ with * is a commutative group:

*	a	b	c	d
a		d		
b				a
c	a			
d				

8. (12 pts) Determine whether the following sets H are subgroups of the given groups G with operations *

a. $H = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}^*$, i.e., H is the set of real numbers of the form $a + b\sqrt{2}$ where a and b are integers not both zero; $G = \mathbb{R}^*$, i.e., $\mathbb{R}^*$ is the set on non-zero real numbers; * is multiplication of real numbers.

b. $H = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in 3\mathbb{Z}\}$; $G = M(2, \mathbb{Z})$; * is matrix addition.