MATH 3360-003 Exam II June 25, 2007

Make Up

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. **For no question (except parts of Question 12) is True/Yes or False/No by itself sufficient as the answer.** <u>Retain</u> this question sheet for your records.

1. Not repeatable

2. (7 pts) Let S be the plane $\mathbb{R}^2$ and for points $p, q \in S$ let $d(p,q)$ denote the (Euclidean) distance between p and q. Let O denote the origin, i.e., $O = (0,0)$. Let ~ be the relation on S given by $p \sim q$ if $d(p,O) = d(q,O)$. Determine whether ~ is an equivalence relation on S.

3A. (7 pts) Let n be a positive integer and let a, b, c, d be integers. Suppose that $a \equiv b (\mathrm{mod}\, n)$ and $c \equiv d (\mathrm{mod}\, n)$. Prove that $ac \equiv bd (\mathrm{mod}\, n)$.

3B. (12 pts) For each of the following pairs of integers find the greatest common divisor and the least common multiple:

a. {748,1360} b. {480,756}

4. (7 pts) For the following pair of integers find the greatest common divisor by using the Euclidean algorithm:

a. {332,544}

5. (7 pts) Prove or disprove: $(a,b) \neq 1$ and $(b,c) \neq 1 \Rightarrow (a,c) \neq 1$

6. (6 pts) Calculate the following value $([22] \oplus [15])^2 \odot [11]$ in:

a. $\mathbb{Z}_5$ b. $\mathbb{Z}_{13}$ c. $\mathbb{Z}_{18}$

(Write the results in terms of the standard congruence classes [0], [1], . . . , [n-1] for $\mathbb{Z}_n$.)

7. (7 pts) Consider the group $G = M_{(\overline{\text{X}})} \times \mathbb{Z}_4$ and the subgroup $H = \langle (\mu_{180}, [1]) \rangle$, where the group $M_{(\overline{\text{X}})}$ is the set of invariant mappings (isometries) of the hour glass and μ_{180} is the 180° rotation about the center of the hour glass. Identify all of the right cosets of H in G.

8. (6 pts) Calculate each of the following:

a. $\left[S_5 : < (1\,234) > \right]$ b. $\left[\mathbb{Z}_4 \times \mathbb{Z}_{12} : < ([2],[2]) > \right]$

c. $\left[\mathbb{Z}_6 \times \mathbb{Z}_{12} : < [2] >_{\mathbb{Z}_6} \times < [2] >_{z_{12}} \right]$

9. (7 pts) Construct the subgroup lattice of $\mathbb{Z}_{24}$.

10. (6 pts)
a. In S_4 find < (1 4 3 2), (2 4) >
b. In $\mathbb{Z}_{24}$ find < [4], [10] >

11. (7 pts) Prove: If $A,\ B$ are abelian groups, then $A \times B$ is an abelian group.

12. (12 pts) Determine whether the following pairs of groups are isomorphic or not. If the pair of groups is isomorphic, you need only state so. If the pair of groups is not isomorphic, you must give a valid reason why the pair of groups is not isomorphic.

a. $\mathbb{Z}_3 \times \mathbb{Z}_6$, $\mathbb{Z}_{18}$ b. $\mathbb{Z}_2 \times \mathbb{Z}_5$, $\mathbb{Z}_{10}$

c. $< (3\,5\,7) >, G$ d. $\mathbb{Z}$, E

e. $\mathbb{Z}_2 \times \mathbb{Z}_4$, $M_{(\square)}$ f. $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$, $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

In part c. the group $< (3\,5\,7) >$ is the subgroup of S_7 generated by the permutation $(3\,5\,7)$ and the group G is the group with operation given by the Cayley table

*	a	b	c
a	c	a	b
b	a	b	c
c	b	c	a

In part d. the group E is the subgroup of $\mathbb{Z}$ of even integers.
In part e. the group $M_{(\square)}$ is the set of invariant mappings (isometries) of the square.

13. (6 pts) Consider Abelian groups of order 48. Give a representative for each possible isomorphism class.