MATH 3360-001 Exam I June 11, 2007

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. (12 pts) Complete each of the following statements:

a. Let $\alpha : S \to T$. α is onto if and only if . . . .

b. Let $\alpha : S \to T$. α is invertible if and only if . . . .

c. Let S be a non-empty set and * an operation on S. $e \in S$ is an identity element for * if and only if . . . .

d. Let (G,*) be a group. A subset H of G is a subgroup of G if and only if . . . .

2. (12 pts) Consider the following sets *S* and equations, each of which defines a rule * for *S*. In each case, determine whether * defines an operation on *S*. If * does not define an operation on *S*, state why not.

a. $S = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| \, a,b,c \in \mathbb{Z} \right\}$, * is matrix multiplication

b. $S = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} \middle| \, a,b,c \in \mathbb{Z} \right\}$, * is matrix multiplication

3. (12 pts) Consider the following equation, which defines an operation * on $\mathbb{Z}$:
$m * n = m + n + mn$

a. Determine if * is associative. If not, state why not.

b. Determine if * has an identity. If not, state why not.

c. Determine if * is commutative. If not, state why not.

4. (12 pts) Complete the following Cayley table so that $G = \{a, b, c, d\}$ with * is a commutative group:

*	a	b	c	d
a		d		
b				a
c	a			
d				

5. (18 pts) Consider the following sets G with associative operations *. In each case, determine whether (G, *) is a group. If (G,*) is not a group, state why it is not.

a. $G = \{0,\ 1,\ 2,\ 3,\ 4,\ \cdots\}$, * is addition of integers

b. $G = \mathbb{Q} \setminus \{0\}$, i.e., G is the set of non-zero rational numbers, * is multiplication of rational numbers

c. $G = M(\mathbb{Z})$, i.e., G is the set of mappings from $\mathbb{Z}$ to $\mathbb{Z}$, * is addition of functions

6. (12 pts) Write each of the following as a single cycle or as a product of disjoint cycles:

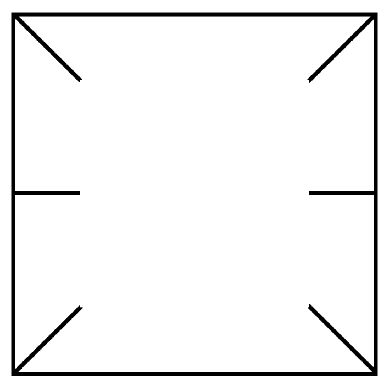
a. $\begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,5\,6\,1\,2\,4 \end{pmatrix}$

b. $(1\ \ 3\ \ 2\ \ 6)\,(2\ \ 5\ \ 1)\,(3\ \ 1\ \ 7\ \ 2)$

c. $(2\ \ 5\ \ 4\ \ 1)^{-1}\,(1\ \ 3\ \ 5\ \ 2)\,(1\ \ 4\ \ 3)$

7. (12 pts) It is known that $\mathbb{R}^2$ with component-wise addition (i.e., $(a,b)+(c,d)=(a+c,b+d)$) forms a group. Show that the subset H = $\{(x,y) \mid x=y,\ x,y \in \mathbb{R}\}$ is a subgroup of $\mathbb{R}^2$.

8. (12 pts) Identify the symmetry group of the figure to the right, i.e., construct a list (or a table) which identifies each of the elements which belong to the symmetry group of the figure. The figure consists of a square centered at the origin minus 4 slits of equal length on the diagonals and 2 slits of equal length on the x-axis.



Also, identify which elements in the symmetry group of the figure are self-inversive, i.e., which elements in the symmetry group of the figure are their own inverses.