MATH 3360-001 Exam II June 24, 2005

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

1. (7 pts) Let S be the plane $\mathbb{R}^2$ and let ~ be the relation on S given by $(a_1, b_1) \sim (a_2, b_2)$ if $a_1 - a_2 = b_1 - b_2$ Determine whether ~ is an equivalence relation on S.

2. (7 pts) Prove: $a \mid b$ and $b \mid c \Rightarrow a \mid c$

3. (12 pts) For each of the following pairs of integers find the greatest common divisor and the least common multiple:

   a. {240,448} b. {264,396}

4. (7 pts) For the following pair of integers write the greatest common divisor as a linear combination of the given integers:

   a. {240,448}

5. (7 pts) Prove or disprove: $a \mid bc \Rightarrow a \mid b$ or $a \mid c$

6. (6 pts) Calculate the following value $([22] \oplus [15])^2 \odot [11]$ in:

   a. $\mathbb{Z}_6$ b. $\mathbb{Z}_{13}$ c. $\mathbb{Z}_{17}$

   (Write the results in terms of the standard congruence classes [0], [1], . . . , [n-1] for $\mathbb{Z}_n$.)

7. (7 pts) Consider the group $G = S_3 \times \mathbb{Z}_3$ and the subgroup $H = \langle ((1\,2), [1]) \rangle$. Identify all of the right cosets of H in G.

8. (6 pts) Calculate each of the following:

a. $[S_4 : <(1\ 2\ 3\ 4)>]$ b. $[\mathbb{Z}_6 \times \mathbb{Z}_8 : <([2],[2])>]$

c. $[\mathbb{Z}_6 \times \mathbb{Z}_8 : <[2]> \times <[2]>]$

9. (8 pts) Construct the subgroup lattice of $\mathbb{Z}_{24}$.

10. (9 pts)
a. In S_8 find < (1 5 4), (2 7) >
b. In $\mathbb{Z}_{20}$ find < [6], [15] >
c. In $\mathbb{Z}_{24}$ find < [6], [15] >

11. (7 pts) Prove: If A, B are abelian groups, then $A \times B$ is an abelian group.

12. (12 pts) Determine whether the following pairs of groups are isomorphic or not. If the pair of groups is isomorphic, you need only state so. If the pair of groups is not isomorphic, you must give a valid reason why the pair of groups is not isomorphic.

a. $\mathbb{Z}_3 \times \mathbb{Z}_8$, $\mathbb{Z}_{24}$ b. $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_5^{\#}$

c. $\mathbb{Z}_2 \times <(1\ 2\ 3)>$, $M_{(\triangle)}$ d. $\mathbb{Z}$, D

e. $\mathbb{Z}_2 \times \mathbb{Z}_2$, $M_{(\square)}$ f. $\mathbb{Z}_3 \times \mathbb{Z}_8$, $\mathbb{Z}_4 \times \mathbb{Z}_6$

In part c. the group $<(1\ 2\ 3)>$ is the subgroup of S_3 generated by the permutation $(1\ 2\ 3)$ and the group $M_{(\triangle)}$ is the set of isometries of the equilateral triangle.

In part d. the group D is the set of diagonal 2×2 matrices with integer entries, i.e., $D = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}$.

In part e. the group $M_{(\square)}$ is the set of isometries of the rectangle.

13. (6 pts) Consider Abelian groups of order 54. Give a representative for each possible isomorphism class.