MATH 3360-001 Exam I June 13, 2005

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. . **For no question is True/Yes or False/No by itself sufficient as the answer**. Retain this question sheet for your records.

1. (12 pts) Complete each of the following statements:

a. Let $\alpha : S \to T$. α is one-to-one if and only if . . .

b. Let $\alpha : S \to T$. β is an inverse of α if and only if . . .

c. Let G be a set with operation *. $(G, *)$ is a group if and only if . . .

d. Let $(G, *)$ be a group. $(G, *)$ is a nonabelian group if and only if . . .

2. (12 pts) Consider the following equations, each of which defines a rule *. In each case, determine whether * defines an operation on $\mathbb{Z}$. If * does not define an operation on $\mathbb{Z}$, state why it does not.

a. $$m * n = \frac{m(n+1)+n(m+1)}{2}$$

b. $$m * n = \frac{(m+(n+1))-((m-1)-n)}{2}$$

3. (12 pts) Consider the following equation, which defines an operation * on $\mathbb{Z}$: $m * n = m + n^2$

a. Determine if * is associative. If not, state why it is not.

b. Determine if * is commutative. If not, state why it is not.

c. Determine if * has an identity. If it does, state what is it?

4. (10 pts) Complete the following Cayley table so that * is a commutative operation with identity and each element has an inverse.

*	a	b	c	d	e
a				c	
b				d	
c	e				d
d					
e	b				

Hint: Start by determining which element must be the identity.

5. (18 pts) Consider the following sets *G* with associative operations *. In each case, determine whether (*G*, *) a group. If (*G*, *) is not a group, state why it is not.

a. $G = \{e^t : t \in \mathbb{R}\}$, * is real multiplication

b. $G = \{\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} : a \in \mathbb{R}\}$, * is matrix multiplication

c. $G = \{\begin{bmatrix} 0 & 0 \\ 0 & a \end{bmatrix} : a \in \mathbb{R}\}$, * is matrix multiplication

6. (12 pts) Write each of the following permutations as a single cycle or a product of disjoint cycles.

a. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 5 & 4 & 2 & 6 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 5 & 4 & 2 & 1 & 7 \end{pmatrix}$

b. $(2\ \ 4\ \ 1\ \ 5)(3\ \ 5\ \ 4)^{-1}(1\ \ 3\ \ 2)$

7. (12 pts) Consider the following groups (*G*, *) and subsets *H*. In each case, determine whether *H* is a subgroup of (*G*, *). If *H* is not a subgroup of (*G*, *), state why it is not.

a. (*G*, *) = $(\mathbb{Q}^+, \times)$, $H = \{2^n : n \in \mathbb{Z}\}$

b. (*G*, *) = $(S_4, \circ)$, *H* = { (1), (1 4), (1 3) , (1 4 3) }

8. (12 pts) Identify the symmetry group for the figure to the right (which is a square surrounded by four congruent isosceles triangles), i.e., construct a list (or a table) which identifies each of the elements which belong to the symmetry group of the figure.

Also, identify which elements in the symmetry group of the figure are self-inversive, i.e., which elements in the symmetry group of the figure are their own inverses.