MATH 3360-001 Exam II June 20, 2003

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question (except question 7) is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. (6 pts)

a. Let S be a non-empty set and ~ a relation on S. Complete the following statement: ~ is symmetric if and only if . . . .

b. Let G with operation * and H with operation # be groups. Complete the following statement: H is a homomorphic image of G if and only if . . . .

2. (6 pts) Find the greatest common divisor of the following pairs of integers:

a. (36,124) b. (48,354)

3. (8 pts) For each of the following pairs of integers write the greatest common divisor as a linear combination of the given pair of integers:

a. (36,124) b. (48,354)

4. (6pts) Calculate the following values in Z_7. (Write the results in terms of the standard congruence classes [0], [1], . . . , [6].)

a. $([44] \oplus [31])^2$ b. $([34] \oplus [28]) \odot [38]$

5. (6 pts) Calculate the following values in Z_{13}. (Write the results in terms of the standard congruence classes [0], [1], . . . , [12].)

a. $([44] \oplus [31])^2$ b. $([34] \oplus [28]) \odot [38]$

6. (6 pts) Prove: If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$

7. (8 pts) Determine which of the following statements are true and which are false. (Answer only T or F).

a. $(a,c) = 1$ and $(b,c) = 1 \Rightarrow (ab,c) = 1$
b. $a \equiv b \pmod{n} \Rightarrow a^2 \equiv b^2 \pmod{n}$
c. $(ab,c) = 1 \Rightarrow (a,c) = 1$ and $(b,c) = 1$
d. $a^2 \equiv b^2 \pmod{n} \Rightarrow a \equiv b \pmod{n}$

8. (6 pts) Let G be an additive group and let $a,b \in G$. Prove that the equation $a + x = b$ always has a solution in G. Find the solution.

9. (6 pts) Let G be a group and let a be a fixed element of G. Define $\lambda : G \to G$ by $\lambda(x) = xa$. Prove that $\lambda \in \mathrm{Sym}(G)$.

10. (6 pts)
a. In S_7 find $\langle (1\ 2\ 4), (3\ 5) \rangle$
b. In Z_{24} find $\langle [6], [21] \rangle$
c. In Z_{25} find $\langle [6], [21] \rangle$

11. (8 pts) Find all of the subgroups of $Z_2 \times Z_4$. Construct the subgroup lattice of $Z_2 \times Z_4$.

12. (6 pts) Assume that H = { u, v, w, x, y, z } is a multiplicative group and that $\theta : S_3 \to H$ is an isomorphism. with

$\theta(\,(1)\,) = x$	$\theta(\,(1\ 2\ 3)\,) = y$	$\theta(\,(1\ 3\ 2)\,) = z$
$\theta(\,(1\ 2)\,) = u$	$\theta(\,(1\ 3)\,) = v$	$\theta(\,(2\ 3)\,) = w$

Find the value in H of:

a. uz b. y^5 c. uzu^{-1}

13. (8 pts) Prove that if G, H and K are groups and if $\theta : G \to H$ and $\phi : H \to K$ are isomorphisms, then $\phi \circ \theta : G \to K$ is an isomorphism.

14. (8 pts) Each of the following pairs of groups are not isomorphic. Give a valid reason for each pair as to why the two groups are not isomorphic.

a. $Z_3 \times Z_4$, $Z_2 \times Z_6$ b. $Z_2 \times Z_2$, $\langle (1\ 2\ 3\ 4) \rangle$
c. $Z_6 \times Z_6$, $S_3 \times S_3$ d. Q, + , $\mathbb{R}$, +

15. (8 pts) Consider Abelian groups of order 48. Give a representative for each possible isomorphism class.