MATH 3360-001 Exam I June 9, 2003

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question (except question 4) is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1a. (4 pts) Let S be a non-empty set and α a map from S $\to$ S. Complete the following statement: α is one-to-one if and only if . . . .

1b. (4 pts) Let S be a non-empty set and * an operation on S. Complete the following statement: e $\in$ S is an identity element for * if and only if . . . .

2. (8 pts) Let α, β be mappings from $\mathbb{Z}$ to $\mathbb{Z}$ defined by α (n) = 3n + 1 and β (n) = n(n+3).

   a. Which of α, β, if any, is onto?
   b. Which of α, β, if any, is one-to-one?

3. (8 pts) Let α, β be mappings from $\mathbb{Z}$ to $\mathbb{Z}$ defined by α (n) = 3n + 1 and β (n) = n(n+3).

   a. Find $\alpha \circ \beta$
   b. Find $\beta \circ \alpha$

4. (8 pts) For a mapping α : S $\to$ S determine which of the following statements are true and which are false.

   a. α is invertible only if α is onto.
   b. α is invertible if α is onto.
   c. A sufficient condition for α to be onto is that it be invertible.
   d. A necessary condition for α to be onto is that it be invertible.

5. (8 pts) Which of the following, if any, define an operation on $\mathbb{Z}$?

   a. $m*n = \frac{m(m+1)}{2} + \frac{n(n-1)}{2}$
   b. $m*n = \frac{m(m+1)}{2} \bullet \frac{n(n-1)}{2}$

6. (10 pts) Consider the following operation * defined on $\mathbb{Z}$: m*n = (m+1)*(n+1).

   a. Is * associative?
   b. Is * commutative?
   c. Does there exist an identity element for *?

7. (10 pts) Complete the following table in such a way that * is commutative and has an identity element and each element has an inverse.

*	a	b	c	d
a	c			b
b	d	a		
c				
d				a

8. (10 pts) Determine whether the given set with given operation forms a group.

a. set = $\mathbb{Z}$, operation = integer multiplication

b. set = D(2, $\mathbb{R}$) = $\left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in \mathbb{R}, a \neq 0, b \neq 0 \right\}$, operation = matrix multiplication

9. (10 pts) Write the following as a single cycle or as a product of disjoint cycles:

a. $\begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,5\,6\,1\,2\,4 \end{pmatrix}$

b. (1 3 2 6) (2 5 1) (3 1 7 2)

c. $(2\ \ 5\ \ 4\ \ 1)^{-1}$ (1 3 5 2) (1 4 3)

10. (10 pts) Consider the set of mappings M(Z) = { f : Z $\rightarrow$ Z }. The set M(Z) with addition as the operation is a group. Determine whether the following subsets of M(Z) are subgroups of M(Z):

a. G_1 = { f $\in$ M(Z) | f(1) = 0 }
b. H_1 = { f $\in$ M(Z) | f(1) = 1 }

11. (10 pts) Find the symmetry group of the figure to the right (three equal length, equally distributed, spokes emanating from a common point). Also, construct the Cayley table for the symmetry group.