MATH 3360 Exam II August 9, 1994

Show your work for each problem. You do <u>not</u> need to rewrite the statements of the problems on your answer sheets.

1. Let $(1\ 2\ 4) \in S_4$. (a) Find $|<(1\ 2\ 4)>|$. (b) Find $[S_4:<(1\ 2\ 4)>]$.
   (c) Find $<(1\ 2\ 4)>(1\ 3\ 2)$.

2. Let G be a finite group and let $a,b \in G$. Prove $o(a^{-1}ba) = o(b)$.

3. Construct a complete subgroup lattice for $\mathbf{Z}_{75}$.

4. Prove $(\mathbf{Z}_4, \oplus) \approx (\mathbf{Z}_5^{\#}, \odot)$.

5. Prove or disprove: $\mathbf{Z}_4\times\mathbf{Z}_2 \approx \mathbf{Z}_2\times\mathbf{Z}_2\times\mathbf{Z}_2$.

6. Which of the following subsets S are subrings of the given rings R?
   (a) $R = M(2,\mathbf{Z})$ with the usual matrix addition and matrix multiplication.
   $S = \{ M \in M(2,\mathbf{Z}) \mid \det(M) = 0 \}$
   (b) $R = \mathbf{Z}[x]$ with the usual polynomial addition and polynomial multiplication.
   $S = \{ p \in \mathbf{Z}[x] \mid p(0) = 0 \}$.

7. Find the characteristic of R. (a) $R = \mathbf{Z}_3\times\mathbf{Z}_4$. (b) $R = \mathbf{Z}_3\times\mathbf{Z}_6$.

8. Prove: If $a > b$ and $b > c$, then $a > c$.

9. Suppose $\theta : \mathbb{C} \to \mathbb{C}$ is a ring isomorphism such that $\theta(x) = x$ for all $x \in \mathbb{R}$. Suppose θ is <u>not</u> the identity mapping. Prove $\theta(i) = -i$. (Hint: Suppose that $\theta(i) = a + bi$ and then use the fact that θ preserves both of the ring operations.)