MATH 3360 Exam II April 3, 1995

Show your work for each problem. You do <u>not</u> need to rewrite the statements of the problems on your answer sheets.

1. Find $d = (42,315)$ and write d as a linear combination of 42 and 315.

2. Prove or disprove: If $(a,b) = 1$ and $(b,c) = 1$, then $(a,c) = 1$.

3. Prove or disprove: $(\mathbf{Z}_6^{\#}, \mathsf{u})$ is a group.

4. Prove that if A and B are Abelian groups, then $A \times B$ is also Abelian.

5. Complete the following table so as to get a group:

$*$	a	b	c
a			
b			
c		b	

6. Find the subgroup <(2 5)(1 3 4)> of S_5 . Find *<(2 5)(1 3 4)>*. Find the index of <(2 5)(1 3 4)> in S_5, i.e., find [S_5 : <(2 5)(1 3 4)>].

7. Identify all of the subgroups of $\mathbf{Z}_{24}$. Construct a complete subgroup lattice for $\mathbf{Z}_{24}$.

8. Let $M_{(a)}$ denote the group of symmetries of the equilateral triangle, i.e., $M_{(a)} = \{ \mu_0, \mu_{120}, \mu_{240}, \mu_{a0}, \mu_{b0}, \mu_{c0} \}$. Find the right cosets in $M_{(a)}$ determined by the subgroup $<\mu_{a0}>$.

9. Identify as true or false. If false, give a reason why.

   (a) $\mathbf{Z}_3 \times \mathbf{Z}_4 \ . \ \mathbf{Z}_{12}$ (b) $\mathbf{Z}_2 \times \mathbf{Z}_6 \ . \ \mathbf{Z}_{12}$ (c) $\mathbf{Z}_3 \times \mathbf{Z}_4 \ . \ \mathbf{Z}_2 \times \mathbf{Z}_6$

10. Identify two non-isomorphic groups of order 18.