MATH 3360 Exam I July 25, 1994

Show your work for each problem. You do <u>not</u> need to rewrite the statements of the problems on your answer sheets. Each problem part will be weighted at the same value.

1. For each ordered pair of integers (a,b) let $\alpha_{(a,b)} : \mathbf{Z} \to \mathbf{Z}$ be given by $\alpha_{(a,b)}(n) = an + b$.

   (i) For which (a,b) is $\alpha_{(a,b)}$ one-to-one?
   (ii) For which (a,b) is $\alpha_{(a,b)}$ onto?

2. Complete the Cayley table so that the operation * is commutative, has an identity and each element has an inverse.

*	w	x	y	z
w	y			x
x	z	w		
y				
z				w

3. Determine whether the set G and the operation * form a group. If so, specify the identity. If not, state a reason why.

   a) $G = \mathbf{Q}^+$, * = (real) multiplication
   b) $G = 2\mathbf{Z}$, * = (real) addition
   c) $G = 2\mathbf{Z}$, * is defined by $m*n = mn/2$.

4. Determine whether the given subset H is a subgroup of the specified group $(G,*)$.

   a) $G = S_5$, * = composition, $H = \{ (1), (1\ 3), (3\ 5), (1\ 5) \}$
   b) $G = \{ \alpha_{(a,b)} \mid \alpha_{(a,b)}(n) = an + b,\ \ a = 1, b \in \mathbf{Z} \}$, * = composition,
   $H = \{ \alpha_{(a,b)} \in G \mid b \in 4\mathbf{Z} \}$

5. Find the symmetry group for a rhombus.

6. For $a,b > 0$ prove or disprove each of the following:

   a) If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.
   b) If $a^2 \equiv b^2 \pmod{n}$, then $a \equiv b \pmod{n}$.

7. Prove: If a, b and c are integers, with $a \mid bc$ and $(a,b) = 1$, then $a \mid c$.