MATH 3360 Exam II March 29, 2000

Answer the problems on separate paper. You do <u>not</u> need to rewrite the problem statements on your answer sheets. Do your own work. ***<u>Show</u>*** all relevant steps which lead to your solutions. Retain this question sheet for your records.

Notation:

$\mathbf{Z} = \{ n : n \text{ is an integer} \}$ $\mathbb{R} = \{ x : x \text{ is a real number} \}$

1. Find the symmetry group for the following figure, which consists of an inscribed regular octagon with two diagonals.

2. Let S be the plane $\mathbb{R}^2$ and let ~ be the relation on S given by $(a_1 , b_1) \sim (a_2 , b_2)$ if $a_1 \cdot b_2 = a_2 \cdot b_1$. Determine whether ~ is an equivalence relation on S.

3. Let a, b, c be integers with $c > 1$ and let n be a fixed integer, $n > 1$. Prove or disprove:

   a) $a \equiv b \pmod n \Rightarrow ac \equiv bc \pmod n$
   b) $ac \equiv bc \pmod n \Rightarrow a \equiv b \pmod n$

4. Prove: $(a,b) = 1$ and $c \mid a \Rightarrow (c,b) = 1$.

5. Find (84,324). Find [84,324].

6. Let H be the subgroup of S_6 given by $H = \langle (16)(235) \rangle$. Find H. Find $|H|$. Find the index of H in S_6, i.e., find $[S_6 : H]$.

7. Find all of the subgroups of $\mathbf{Z}_{30}$ and construct a subgroup lattice for $\mathbf{Z}_{30}$.

8. Assume that G is a finite group such that $|G| < 32$. Assume that H is a subgroup of G such that $|H| > 2$ and $[G : H] > 9$. Find $|G|$, $|H|$ and $[G : H]$.

9. Find the right cosets of $\langle [6] \rangle$ in $\mathbf{Z}_{24}$.

10. Determine whether the following are true or false:

    a) $\mathbf{Z}_3 \times \mathbf{Z}_4 \approx \mathbf{Z}_2 \times \mathbf{Z}_6$
    b) $\mathbf{Z}_6 \times \mathbf{Z}_5 \approx \mathbf{Z}_3 \times \mathbf{Z}_{10}$