

Name _____

Score _____

In-class Exam

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

Part I. (18 pts) Do three (3) of the following four problems:

1. Find the Laplace transform of $f(t) = \begin{cases} 1-t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$
2. Find the Laplace transform of $f(t) = (5t^2 - e^{-4t})^2$
3. Find the Laplace transform of $f(t) = te^{4t} \cos 7t$
4. Find the Laplace transform of $f(t) = (3t^2 + e^{-4t})u(t-5)$

Part II. (18 pts) Do three (3) of the following four problems:

5. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\left(\frac{4}{s} - \frac{3}{s^2}\right)^2\right)$
6. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{4s-15}{s^2+40}\right)$
7. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left(\frac{7s-9}{s^2-8s+25}\right)$
8. Find the following inverse Laplace transform $\mathcal{L}^{-1}\left\{\frac{e^{-7s}}{(s-3)(s+4)}\right\}$

Part III. (50 pts) Do five (5) of the following six problems:

9. Use the method of Laplace transforms to solve the following linear differential equation
$$y' + 5y = -3$$
$$y(0) = -2$$
10. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 8y' + 15y = 0$$
$$y(0) = -2, y'(0) = 3$$
11. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 8y' + 16y = 6$$
$$y(0) = -1, y'(0) = 1$$
12. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 6y' + 8y = u(t - 5)$$
$$y(0) = 0, y'(0) = 0$$
13. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' + 16y = \cos 2t$$
$$y(0) = 0, y'(0) = 3$$
14. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' + 25y = \delta(t - 4)$$
$$y(0) = 2, y'(0) = -4$$

Name _____

Score _____

Take-Home Exam

Answer the problems on this sheet of paper. Work carefully. Do your own work. **Show all relevant supporting steps!**

15. (6 pts) A mass weighing 20 lbs is attached to a spring and stretches the spring 4 feet. The mass is released at a point 3 inches above the equilibrium position with an initial downward velocity of 6 inches/sec. Find the equation of motion.

Answer the next problem on the back of the sheet

16. (10 pts) A mass weighing 9 lbs is attached to a 6 ft spring and stretches the spring so that its new length is 9 ft. This mass is replaced by another mass which weighs 8 lbs. The medium through which the spring-mass system moves creates a damping force equal to twice the instantaneous velocity. The mass is released at a point 2 ft below the equilibrium position with an initial downward velocity of 1 ft/s. Find the equation of motion.