Case III: $(q = 1)$

Here, the series is $\sum_{k=1}^{\infty} \frac{\ln k}{k}$, which diverges by the integral test:

$$\int_1^{\infty} \frac{\ln x}{x}\,dx = \lim_{b\to\infty}\left[\frac{\ln^2 x}{2}\right]\Bigg|_1^b = \infty$$

Let $u = \ln x$; $du = \frac{1}{x}\,dx$

8.4 PROBLEM SET

A *The most common series used for comparison are given in Problems 1 and 2. Tell when each converges and when it diverges.*

1. geometric series: $\sum_{k=0}^{\infty} r^k$ **2.** p-series: $\sum_{k=1}^{\infty} \frac{1}{k^p}$

Each series in Problems 3–12 can be compared to the geometric series or p-series given in Problems 1 and 2. State which, and then determine whether it converges or diverges.

3. $\sum_{k=1}^{\infty} \cos^k\left(\frac{\pi}{6}\right)$ **4.** $\sum_{k=0}^{\infty} 0.5^k$

5. $\sum_{k=0}^{\infty} 1.5^k$ **6.** $\sum_{k=0}^{\infty} 2^{k/2}$

7. $\sum_{k=1}^{\infty} \frac{1}{k}$ **8.** $\sum_{k=1}^{\infty} \frac{1}{k^{0.5}}$

9. $\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$ **10.** $\sum_{k=1}^{\infty} \sqrt{\frac{2}{k}}$

11. $\sum_{k=0}^{\infty} 1^k$ **12.** $\sum_{k=1}^{\infty} e^k$

Test the series in Problems 13–44 for convergence.

13. $\sum_{k=1}^{\infty} \frac{1}{k^2+k}$ **14.** $\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$

15. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ **16.** $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$

17. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{2k+3}}$ **18.** $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$

19. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3+2}}$ **20.** $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2+1}}$

21. $\sum_{k=1}^{\infty} \frac{2k^2}{k^4-4}$ **22.** $\sum_{k=1}^{\infty} \frac{k+1}{k^2+1}$

23. $\sum_{k=1}^{\infty} \frac{(k+2)(k+3)}{k^{7/2}}$ **24.** $\sum_{k=1}^{\infty} \frac{(k+1)^3}{k^{9/2}}$

25. $\sum_{k=1}^{\infty} \frac{2k+3}{k^2+3k+2}$ **26.** $\sum_{k=1}^{\infty} \frac{3k^2+2}{k^2+3k+2}$

27. $\sum_{k=1}^{\infty} \frac{k}{(k+2)2^k}$ **28.** $\sum_{k=1}^{\infty} \frac{5}{4^k+3}$

29. $\sum_{k=1}^{\infty} \frac{1}{k(k+2)}$ **30.** $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

31. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}\,2^k}$ **32.** $\sum_{k=1}^{\infty} \frac{1,000}{\sqrt{k}\,3^k}$

33. $\sum_{k=1}^{\infty} \frac{|\sin(k!)|}{k^2}$ **34.** $\sum_{k=2}^{\infty} \frac{1}{\sqrt{k}\,\ln k}$

35. $\sum_{k=1}^{\infty} \frac{2k^3+k+1}{k^3+k^2+1}$ **36.** $\sum_{k=1}^{\infty} \frac{6k^3-k-4}{k^3-k^2-3}$

37. $\sum_{k=1}^{\infty} \frac{k}{4k^3-5}$ **38.** $\sum_{k=1}^{\infty} \frac{\ln k}{\sqrt{2k+3}}$

39. $\sum_{k=1}^{\infty} \frac{k^2+1}{(k^2+2)k^2}$ **40.** $\sum_{k=1}^{\infty} \sin\frac{1}{k}$

41. $\sum_{k=1}^{\infty} \frac{6k^2+2k+1}{k^{1.1}(4k^2+k+4)}$ **42.** $\sum_{k=1}^{\infty} \frac{6k^2+2k+1}{k^{0.9}(4k^2+k+4)}$

43. $\sum_{k=1}^{\infty} \frac{\sqrt[6]{k}}{\sqrt[4]{k^3+2}\,\sqrt[8]{k}}$ **44.** $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt[3]{k^3+1}\,\sqrt[6]{k^5}}$

B *Test the series given in Problems 45–52 for convergence.*

45. $\sum_{k=1}^{\infty} \frac{1}{k^3+4}$ **46.** $\sum_{k=2}^{\infty} \frac{\ln k}{k-1}$

47. $\sum_{k=1}^{\infty} \frac{\ln(k+1)}{(k+1)^3}$ **48.** $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$

49. $\sum_{k=2}^{\infty} \frac{1}{(k+3)(\ln k)^{1.1}}$ **50.** $\sum_{k=2}^{\infty} \frac{1}{(k+3)(\ln k)^{0.9}}$

51. $\sum_{k=1}^{\infty} k^{(1-k)/k}$ **52.** $\sum_{k=1}^{\infty} k^{(1+k)/k}$

53. Show that the series

$$\sum_{k=1}^{\infty} \frac{k^2}{(k+3)!} = \frac{1}{4!} + \frac{4}{5!} + \frac{9}{6!} + \cdots$$

converges by using the limit comparison test.

54. Show that the series

$$1 + \frac{1}{1\cdot 3} + \frac{1}{1\cdot 3\cdot 5} + \frac{1}{1\cdot 3\cdot 5\cdot 7} + \cdots + \frac{2^k k!}{(2k+1)!} + \cdots$$

converges. *Hint*: Compare with the convergent series $\sum 1/k!$.