50. Suppose that an oil well produces \( P(t) \) thousand barrels of crude oil per month according to the formula

\[
P(t) = 1000e^{-0.1t} - 100e^{-t}
\]

where \( t \) is the number of months the well has been in production. What is the total amount of oil produced by the oil well?

51. Let \( f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x \geq 1 \\ 1 & \text{for } -1 < x < 1 \\ e^{-x} & \text{for } x \leq -1 \end{cases} \)

Sketch the graph of \( f \) and evaluate

\[
\int_{-2}^{2} f(x) \, dx
\]

52. Find all values of \( p \) for which \( \int_{-1}^{1} \frac{dx}{x^p(\ln x)^p} \) converges, and find the value of the integral when it exists.

53. Find all values of \( p \) for which \( \int_{0}^{1} \frac{dx}{x^p} \) converges, and find the value of the integral when it exists.

54. Find all values of \( p \) for which \( \int_{0}^{1} \frac{dx}{x^p} \) converges, and find the value of the integral when it exists.

55. **Counterexample Problem** Discuss the calculation

\[
\int_{-1}^{1} \frac{dx}{(x^2 + 1)^{1/2}} = \int_{-1}^{1} \frac{dx}{(x^2 + 1)^{1/2}} = -1 \int_{-1}^{1} \frac{dx}{x^2 + 1} = \pm \sqrt{2}
\]

Is the calculation correct? Explain.

56. **Journal Problem** *College Mathematics Journal* □ Peter Lindstrom of North Lake College in Irving, Texas, had a student who handled an \( \infty/\infty \) form as follows:

\[
\int_{0}^{1} \frac{dx}{x(\ln x)^2} = \int_{0}^{1} \frac{e^{-x}}{x} \, dx
\]

What is wrong, if anything, with this student’s solution?

57. Find \( \int_{1}^{e} f(x) \, dx \), where

\[
f(x) = \begin{cases} \frac{1}{x} & \text{for } 0 < x \leq 1 \\ \frac{1}{\sqrt{x}} \frac{1}{(x - 1)^{3/2}} & \text{for } 1 < x < 2 \end{cases}
\]

58. Evaluate the improper polar integral \( \int_{0}^{\infty} \theta e^{-\theta} \, d\theta \).

59. Find the total area between the spirals \( r = e^{-\theta} \) and \( r = e^{-\theta^2} \) for \( \theta \geq 0 \).