$$= \frac{9}{2\sqrt{2}} \sin^{-1}\left[\frac{\sqrt{2}}{3}(x-4)\right] + \frac{x-4}{2}\sqrt{16x-2x^2-23} + C$$

This last step requires back-substituting from θ to u and then from u to x. Details are left as an exercise. ■

7.3 PROBLEM SET

A 1. **WHAT DOES THIS SAY?** Explain how to integrate $\int \sin^m x \cos^n x\, dx$ when m and n are both even.

2. **WHAT DOES THIS SAY?** Explain how to integrate $\int \tan^m x \sec^n x\, dx$ when n is even.

3. **WHAT DOES THIS SAY?** Explain the process of using a trigonometric substitution on integrals of the form $\sqrt{a^2+u^2}$.

4. **WHAT DOES THIS SAY?** Explain the process of using a trigonometric substitution on integrals of the form $\sqrt{a^2-u^2}$. How is this different from handling an integral involving $\sqrt{u^2-a^2}$?

Evaluate the integrals in Problems 5–50.

5. $\int \cos^3 x\, dx$
6. $\int \sin^5 x\, dx$
7. $\int \sin^2 x \cos^3 x\, dx$
8. $\int \sin^3 x \cos^3 x\, dx$
9. $\int \sqrt{\cos t}\, \sin t\, dt$
10. $\int \frac{\cos x\, dx}{1+3\sin x}$
11. $\int e^{\cos x} \sin x\, dx$
12. $\int \cos^2(2t)\, dt$
13. $\int \sin^2 x \cos^2 x\, dx$
14. $\int \frac{\sin x\, dx}{\cos^5 x}$
15. $\int \tan 2\theta\, d\theta$
16. $\int \sec\left(\frac{x}{2}\right) dx$
17. $\int \tan^3 x \sec^4 x\, dx$
18. $\int \sec^5 x \tan x\, dx$
19. $\int (\tan^2 x + \sec^2 x)\, dx$
20. $\int (\sin x + \cos x)^2\, dx$
21. $\int \tan^2 u \sec u\, du$
22. $\int \sec^4 x\, dx$
23. $\int \sqrt[3]{\tan x}\, \sec^2 x\, dx$
24. $\int e^x \sec(e^x)\, dx$
25. $\int x \sin x^2 \cos x^2\, dx$
26. $\int x \sec^2 x\, dx$
27. $\int \tan^4 t \sec t\, dt$
28. $\int \csc(2\theta)\, d\theta$
29. $\int \csc^3 x \cot x\, dx$
30. $\int \csc^2 x \cot^2 x\, dx$
31. $\int \csc^2 x \cos x\, dx$
32. $\int \tan x \csc^3 x\, dx$
33. $\int \sqrt{4-t^2}\, dt$
34. $\int \frac{dx}{\sqrt{9-x^2}}$
35. $\int \frac{x+1}{\sqrt{4+x^2}}\, dx$
36. $\int \sqrt{9+x^2}\, dx$
37. $\int \frac{dx}{\sqrt{x^2-7}}$
38. $\int \frac{dx}{5+2x^2}$
39. $\int \frac{dx}{\sqrt{5-x^2}}$
40. $\int \frac{dx}{x\sqrt{7x^2-4}}$
41. $\int \frac{dx}{x^2\sqrt{4-x^2}}$
42. $\int \frac{dx}{x\sqrt{x^2+9}}$
43. $\int \frac{\sqrt{x^2-4}}{x}\, dx$
44. $\int \frac{dx}{(x-1)^2+4}$
45. $\int \frac{dx}{9-(x+1)^2}$
46. $\int \sqrt{2x-x^2}\, dx$
47. $\int \frac{dx}{\sqrt{x^2-2x+6}}$
48. $\int \frac{dx}{\sqrt{x^2+8x+3}}$
49. $\int \frac{\sin^3 u\, du}{\cos^5 u}$
50. $\int \frac{\sec^2 x\, dx}{\tan^2 x + \sec^2 x}$

B 51. Find the average value of $f(x) = \sin^2 x$ over the interval $[0, \pi]$.

52. Find the centroid of the region (correct to two decimal places) bounded by the curve $y = \cos^2 x$, the x-axis, and the vertical lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$.

53. Find the volume (correct to four decimal places) of the solid generated when the region bounded by the curve $y = \sin^2 x$ and the x-axis is revolved about the y-axis $0 \le x \le \pi$.

54. A particle moves along the x-axis in such a way that the acceleration at time t is $a(t) = \sin^2 t$. What is the total distance traveled by the particle over the time interval $[0, \pi]$ if its initial velocity is $v(0) = 2$ units per second?

55. Evaluate $\int \sqrt{1+\cos x}\, dx$.

Hint: Use the identity $\cos x = 2\cos^2 \frac{x}{2} - 1$.

In Problems 56–59, use the following identities:

$$\sin A \cos B = \tfrac{1}{2}[\sin(A-B) + \sin(A+B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A-B) + \cos(A+B)]$$

56. $\int \sin 3x \sin 5x\, dx$
57. $\int \cos \frac{x}{2} \sin 2x\, dx$
58. $\int \cos 7x \cos(-3x) \sin 4x\, dx$
59. $\int \sin^2 3x \cos 4x\, dx$

60. Let f be a twice differentiable function that satisfies the initial value problem

$$f''(x) = \tfrac{1}{2}(\tan x) f'(x) \qquad f'(0) = f(0) = 1$$

on the interval $\left[0, \frac{\pi}{2}\right]$. Find the arc length of the curve $y = f(x)$ over this interval.

C 61. Evaluate $\int \frac{x\, dx}{9 - x^2 - \sqrt{9-x^2}}$.