

Solution

$$\text{Let } u = \tan^{-1} x \text{ and } dv = dx$$

$$du = \frac{dx}{1+x^2} \quad v = x$$

$$\int_0^{\pi/4} \underbrace{\tan^{-1} x}_u \underbrace{dx}_{dv} = \underbrace{(\tan^{-1} x)}_u \underbrace{x}_v \Big|_0^{\pi/4} - \int_0^{\pi/4} \underbrace{\frac{x dx}{1+x^2}}_{v du}$$

$$= \left[x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) \right] \Big|_0^{\pi/4}$$

Use substitution where $w = 1 + x^2$ and $dw = 2x dx$.

$$\int \frac{x dx}{1+x^2} = \int \frac{dw}{2w} = \frac{1}{2} \ln w = \frac{1}{2} \ln(1+x^2)$$

$$= \left[\frac{\pi}{4} \left(\tan^{-1} \frac{\pi}{4} \right) - \frac{1}{2} \ln \left(1 + \frac{\pi^2}{16} \right) \right] - \left[0 - \frac{1}{2} \ln 1 \right]$$

$$= \frac{\pi}{4} \tan^{-1} \frac{\pi}{4} - \frac{1}{2} \ln \left(1 + \frac{\pi^2}{16} \right) \approx 0.2827$$

Check with an integration table (Formula 457, for example, with $a = 1$). ■

7.2 PROBLEM SET

1 Find each integral in Problems 1–16.

1. $\int x e^{-2x} dx$

2. $\int x \sin x dx$

3. $\int x \ln x dx$

4. $\int x \tan^{-1} x dx$

5. $\int \sin^{-1} x dx$

6. $\int x^2 \sin x dx$

7. $\int e^{-3x} \cos 4x dx$

8. $\int e^{2x} \sin 3x dx$

9. $\int x^2 \ln x dx$

10. $\int (x + \sin x)^2 dx$

11. $\int \sin(\ln x) dx$

12. $\int x \sin x \cos x dx$

13. $\int \ln(x^2 + 1) dx$

14. $\int \sin \sqrt{x} dx$

15. $\int \frac{x e^{-x}}{(x-1)^2} dx$

16. $\int \frac{\ln(\sin x) dx}{\tan x}$

Find the exact value of the definite integrals in Problems 17–22 using integration by parts, and then check by using a calculator to find an approximate answer correct to four decimal places.

17. $\int_1^4 \sqrt{x} \ln x dx$

18. $\int_1^e x^3 \ln x dx$

19. $\int_1^e (\ln x)^2 dx$

20. $\int_{1/3}^e 3(\ln 3x)^2 dx$

21. $\int_0^{\pi} e^{2x} \cos 2x dx$

22. $\int_0^{\pi} x(\sin x + \cos x) dx$

B 23. **WHAT DOES THIS SAY?** Describe the process known as integration by parts.

In Problems 24–27, first use an appropriate substitution and then integrate by parts to evaluate the integral. Remember to give your answers in terms of x .

24. $\int \frac{\ln x \sin(\ln x)}{x} dx$

25. $\int [\sin 2x \ln(\cos x)] dx$

26. $\int e^{2x} \sin e^x dx$

27. $\int [\sin x \ln(2 + \cos x)] dx$

28. a. Evaluate $\int \frac{x^3}{x^2 - 1} dx$ using integration by parts.

b. Evaluate the integral in part a by first dividing the integrand.

29. Evaluate $\int \cos^2 x dx$.

30. Evaluate $\int \frac{x dx}{\sqrt{x^2 + 1}}$.

31. Use Problem 29 to evaluate $\int x \cos^2 x dx$ using integration by parts.

32. Use Problem 30 to evaluate $\int \frac{x^3 dx}{\sqrt{x^2 + 1}}$ using integration by parts.

33. Find $\int x^n \ln x dx$, where n is any positive real number.