19. $\int \dfrac{x^2\, dx}{\sqrt{x^2+1}}$

20. $\int \dfrac{dx}{x^2\sqrt{x^2+16}}$

21. $\int \dfrac{x\, dx}{\sqrt{4x^2+1}}$

22. $\int \dfrac{dx}{x\sqrt{1-9x^2}}$

23. $\int e^{-4x}\sin 5x\, dx$

24. $\int x\sin^{-1}x\, dx$

Evaluate the integrals in Problems 25–33. If you use the integral table, state the number of the formula used, and if you use substitution, show each step. If you use an alternative table of integrals, then cite the source as well as the formula.

25. $\int (1+bx)^{-1}\, dx$

26. $\int \dfrac{x\, dx}{\sqrt{a^2-x^2}}$

27. $\int x(1+x)^3\, dx$

28. $\int x\sqrt{1+x}\, dx$

29. $\int xe^{4x}\, dx$

30. $\int x\ln 2x\, dx$

31. $\int \dfrac{dx}{1+e^{2x}}$

32. $\int \ln^3 x\, dx$

33. $\int \dfrac{x^3\, dx}{\sqrt{4x^4+1}}$

B **SMH** *Use the Student Mathematics Handbook or other available integration tables to find the integrals given in Problems 34–39. Cite the formula number or source you are using.*

34. $\int \dfrac{\sqrt{4x^2+1}}{x}\, dx$

35. $\int \sec^3\left(\dfrac{x}{2}\right)\, dx$

36. $\int \sin^6 x\, dx$

37. $\int \dfrac{dx}{9x^2+6x+1}$

38. $\int (9-x^2)^{3/2}\, dx$

39. $\int \dfrac{\sin^2 x}{\cos x}\, dx$

40. Derive the **sine squared formula** shown on the inside back cover (Formula 348):

$$\int \sin^2 x\, dx = \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + C$$

Hint: Use the identity $\sin^2 x = \dfrac{1-\cos 2x}{2}$.

41. Derive the **cosine squared formula** shown on the inside back cover (Formula 317):

$$\int \cos^2 x\, dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin 2x + C$$

Hint: Use the identity $\cos^2 x = \dfrac{1+\cos 2x}{2}$.

Problems 42–44 use substitution to integrate certain powers of sine and cosine.

42. $\int \sin^4 x\cos x\, dx$ *Hint:* Let $u = \sin x$.

43. $\int \sin^3 x\cos^4 x\, dx$ *Hint:* Let $u = \cos x$.

44. $\int \sin^2 x\cos^2 x\, dx$ *Hint:* Use the identities shown in Problems 40 and 41.

45. Exploration Problem Using Problems 42–44 formulate a procedure for integrals of the form

$$\int \sin^m x\cos^n x\, dx$$
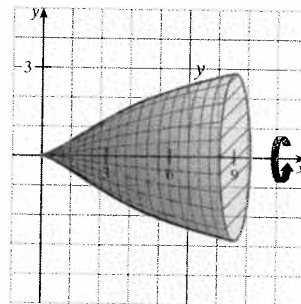
Find each integral in Problems 46–51.

46. $\int \dfrac{e^x\, dx}{1+e^{x/2}}$

47. $\int \dfrac{dx}{x^{1/2}+x^{1/4}}$

48. $\int \dfrac{4\, dx}{x^{1/3}+2x^{1/2}}$

49. $\int \dfrac{18\tan^2 t\sec^2 t}{(2+\tan^3 t)^2}\, dt$

50. $\int \dfrac{dx}{\left(x+\frac{1}{2}\right)\sqrt{4x^2+4x}}$

51. $\int \dfrac{e^{-x}-e^x}{e^{2x}+e^{-2x}+2}\, dx$

52. Find the area of the region bounded by the graphs of $y = \dfrac{2x}{\sqrt{x^2+9}}$ and $y = 0$ from $x = 0$ to $x = 4$.

53. Find the volume of the solid generated when the region under the curve

$$y = \dfrac{x^{3/2}}{\sqrt{x^2+9}}$$

between $x = 0$ and $x = 9$ is revolved about the x-axis.

54. Find the volume of the solid generated when the curve $y = x(1-x^2)^{1/4}$ from $x = 0$ to $x = 1$ is revolved about the x-axis.

55. Find the volume of the solid generated when the curve

$$y = \dfrac{1}{\sqrt{x}}(1+\sqrt{x})^{1/3}$$

between $x = 1$ and $x = 4$ is revolved about the y-axis.

56. Find the volume of the solid generated when the curve $x = \sqrt[4]{4-y^2}$ between $y = 1$ and $y = 2$ is revolved about the y-axis.

57. Let $y = f(x)$ be a function that satisfies the differential equation

$$xy' = \sqrt{(\ln x)^2 - x^2}$$

Find the arc length of $y = f(x)$ between $x = \frac{1}{4}$ and $x = \frac{1}{2}$.

58. Find the arc length of the curve $y = \ln(\cos x)$ on the interval $[0, \pi/4]$.

59. Find the area of the surface generated when the curve $y = x^2$ on the interval $[0,1]$ is revolved about the x-axis.

60. Find the area of the surface generated when the curve $y = x^2$ on the interval $[0,1]$ is revolved about the y-axis.

61. Show that $\int \csc x\, dx = -\ln|\csc x + \cot x| + C$.

Hint: Multiply the integrand by $\dfrac{\csc x + \cot x}{\csc x + \cot x}$.

62. Find $\int 2\sin x\cos x\, dx$ by using the indicated substitution.

a. Let $u = \cos x$.