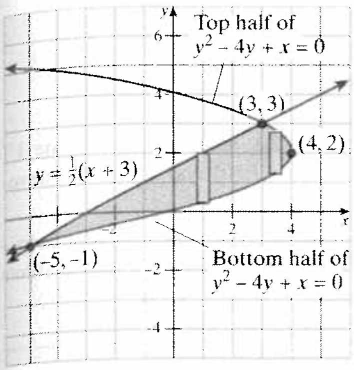


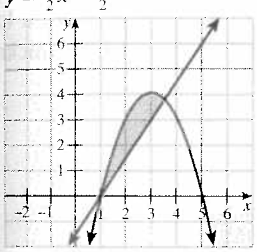
Figure 6.10 The area between the curves $x = 4y - y^2$ and $x = 2y - 3$ using vertical strips

In Example 5, the area can also be found by using vertical strips, but the procedure is more complicated. Note in Figure 6.10 that on the interval $[-5, 3]$, a representative vertical strip would extend from the bottom half of the parabola $y^2 - 4y + x = 0$ to the line $y = \frac{1}{2}(x + 3)$, whereas on the interval $[3, 4]$, a typical vertical strip would extend from the bottom half of the parabola $y^2 - 4y + x = 0$ to the top half. Thus, the area is given by the sum of two integrals, the second of which requires solving the equation $y^2 - 4y + x = 0$ for y using the quadratic formula. With some effort, it can be shown that the computation of area by vertical strips gives the same result as that found by horizontal strips in Example 5.

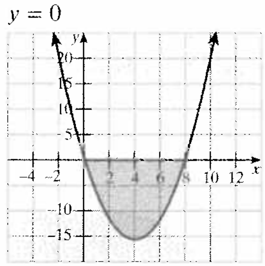
6.1 PROBLEM SET

A *Sketch a representative vertical or horizontal strip and find the area of the given regions bounded by the specified curves in Problems 1–6.*

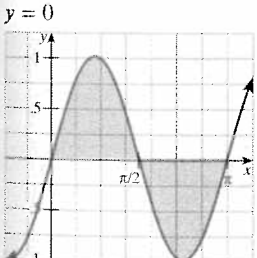
1. $y = -x^2 + 6x - 5$, $y = \frac{3}{2}x - \frac{3}{2}$



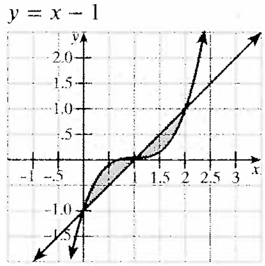
2. $y = x^2 - 8x$, $y = 0$



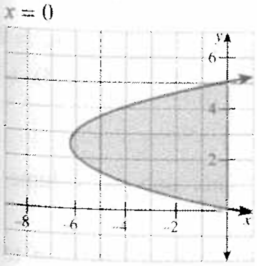
3. $y = \sin 2x$ on $[0, \pi]$, $y = 0$



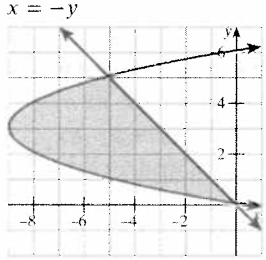
4. $y = (x - 1)^3$, $y = x - 1$



5. $x = y^2 - 5y$, $x = 0$



6. $x = y^2 - 6y$, $x = -y$



Sketch the region bounded between the given curves and then find the area of each region in Problems 7–25.

7. $y = x^2$, $y = x$, $x = -1$, $x = 1$

8. $y = x^3$, $y = x$, $x = -1$, $x = 1$

9. $y = x^2$, $y = x^3$

10. $y = x^2$, $y = \sqrt[3]{x}$

11. $y = x^2 - 1$, $x = -1$, $x = 2$, $y = 0$

12. $y = 4x^2 - 9$, $x = 3$, $x = 0$, $y = 0$

13. $y = x^4 - 3x^2$, $y = 6x^2$

14. $x = 8 - y^2$, $x = y^2$

15. $x = 2 - y^2$, $x = y$

16. $y = x^2 + 3x - 5$, $y = -x^2 + x + 7$

17. $y = 2x^3 + x^2 - x - 1$, $y = x^3 + 2x^2 + 5x - 1$

18. $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{4}$

19. $y = \sin x$, $y = \sin 2x$, $x = 0$, $x = \pi$

20. $y = |x|$, $y = x^2 - 6$

21. $y = |4x - 1|$, $y = x^2 - 5$, $x = 0$, $x = 4$

22. x-axis, $y = x^3 - 2x^2 - x + 2$

23. y-axis, $x = y^3 - 3y^2 - 4y + 12$

24. $y = e^x$, $y = \frac{1}{2}e^x + \frac{1}{2}$, $x = -2$, $x = 2$

25. $y = \dfrac{1}{\sqrt{1 - x^2}}$, $y = \dfrac{2}{x + 1}$, y-axis

B **26. WHAT DOES THIS SAY?** When finding the area between two curves, discuss criteria for deciding between vertical and horizontal strips.

27. Find the number k (correct to two decimal places) so that the line $y = k$ bisects the area under the curve $y = \sin^{-1} x$ for $0 \le x \le 1$.

28. Find the area of the region that contains the origin and is bounded by the lines $2y = 11 - x$ and $y = 7x + 13$ and the curve $y = x^2 - 5$.

29. Show that the region defined by the inequalities $x^2 + y^2 \le 8$, $x \ge y$, and $y \ge 0$ has area π.

30. Find the area of the region bounded by the curve $\sqrt{x} + \sqrt{y} = 1$ and the coordinate axes.