

Name _____

Score _____

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!** Attach this sheet to the front of your answers.

Part I. (18 pts) Do three (3) of the following four problems:

1. Find the Laplace transform of $f(t) = \begin{cases} t+2 & 0 \leq t < 5 \\ -3 & 5 \leq t < \infty \end{cases}$
2. Find the Laplace transform of $f(t) = (e^{-5t} - 3t^2)^2$
3. Find the Laplace transform of $f(t) = te^{-7t} \cos 3t$
4. Find the Laplace transform of $f(t) = (t^2 - 3t)u(t-5)$

Part II. (18 pts) Do three (3) of the following four problems:

5. Find the following inverse Laplace transform $\mathcal{L}^{-1} \left(\left(\frac{6}{s^2} - \frac{5}{s^3} \right)^2 (s+3) \right)$
6. Find the following inverse Laplace transform $\mathcal{L}^{-1} \left(\frac{9s-5}{s^2+7} \right)$
7. Find the following inverse Laplace transform $\mathcal{L}^{-1} \left(\frac{3s+8}{s^2+8s+65} \right)$
8. Find the following inverse Laplace transform $\mathcal{L}^{-1} \left\{ \frac{e^{-3s}}{s^2(s+5)} \right\}$

Part III. (65 pts) Do five (5) of the following six problems:

9. Use the method of Laplace transforms to solve the following linear differential equation
$$y' + 5y = 3t$$
$$y(0) = -4$$
10. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 2y' - 15y = 0$$
$$y(0) = 2, y'(0) = -1$$
11. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 10y' + 25y = t$$
$$y(0) = 0, y'(0) = -2$$
12. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 7y' + 10y = u(t - 4)$$
$$y(0) = 0, y'(0) = -1$$
13. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' + 25y = \cos 4t$$
$$y(0) = 0, y'(0) = -3$$
14. Use the method of Laplace transforms to solve the following linear differential equation
$$y'' - 6y' + 9y = \delta(t - \pi)$$
$$y(0) = 1, y'(0) = -1$$